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DESIGN OF CLASSICAL STRAIGHT LINE MECHANISMS

A THESIS

Presented to

The Faculty of the Graduate Division

by

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In Partial Fulfillment

Of the Requirements for the Degree

Master of Science in Mechanical Engineering

Georgia Institute of Technology

June, 1965

DESIGN OF CLASSICAL STRAIGHT LINE MECHANISMS

Approved:

Date Approved by Chairman: 5/15/65

ACKNOWLEDGMENTS

The author wishes to express his sincere appreciation to all those individuals who have contributed to this work by their help and encouragement. Appreciation is extended to Dr. Joseph P. Vidosic, as faculty advisor, to Dr. Harold L. Johnson and Dr. William J. Lnenicka for their review of this work and their assistance during its development.

Special thanks is extended to Dr. Delbert Tesar who introduced the author to the problem and provided the basic theory upon which the work is based. Acknowledgment is made to the personnel of the Rich Electronic Center for their help with the computer work and to Phillip Knowles and Louis Minor for their assistance in the preparation of the charts.

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SUMMARY

The principal objective of this work was a study of the output of the classical straight line, four-bar mechanisms. These are the Watt, Evans, and symmetrical mechanisms. This investigation was made by analyzing many hundreds of linkages of each type and making a study of the resulting data. Emphasis was placed upon organizing the data into a form which could be used by a designer for synthesis of these mechanisms. The study was based upon existing theories as presented by Delbert Tesar in his Ph. D. thesis (Georgia Institute of Technology, 1964).

In order to obtain the necessary data for the investigation, computer programs were written based upon the existing theories and were used to analyze the linkages to be studied. In this work, the output of interest from these programs was the length of the approximate straight line output and the proportions of the links in the mechanisms. The sizes of all linkages and the lengths of their straight line outputs were scaled to the same relative size so that meaningful results could be obtained from the comparisons. Charts were drawn which showed the length of straight line output within a specified deviation as a function of the design parameters. The presentation of data in this form allowed the easy comparison of the length of straight line output for all the linkages investigated. These charts may also be used by a designer to determine the length of straight line output that a specific linkage will produce, or to determine a linkage which will produce a given length

of straight line output. Examples were used to demonstrate the procedures for designing any of the three types of linkages when a necessary length of straight line output is given.

Of the three types of mechanisms studied, the Watt mechanism produced the most accurate length of straight line output. But, the configurations possible with the Watt mechanism are somewhat limited because the two cranks must be parallel in the design of initial position. The Evans mechanism will produce a longer length of straight line output than the Watt but it is usually not as accurate.

CHAPTER I

INTRODUCTION

The planar four-bar linkage which will generate a given function has been in use for many decades. Although this type of linkage may be analyzed relatively easily, its synthesis proves more difficult. One special type of function which may be generated with a four-bar mechanism is a straight line. Of course the slider crank mechanism accomplishes this task, but pinned linkages have definite desirable properties which make them particularly attractive for this use. Some of these properties are:

(a) The proper lubrication of the cylindrical bearings in a pinned linkage is easy to accomplish, and therefore friction and wear can be reduced to a very low level. In the slider crank, lubrication and wear at the point of sliding contact are problems.

(b) The four-bar linkage is light, simple, and easily manufactured with high accuracy using standard machine tools. When manufacturing errors do occur, the effects on the operation of the linkage are very small. Investigation by D. Tesar[1*] shows that a 0.5% error in manufacture will produce a reduction in accuracy of straight line output over a given distance from $\pm 1.0\times10^{-6}$ to $\pm 5.0\times10^{-6}$.

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* Superscripts refer to items cited in the bibliography.

(c) Since the four-bar linkage has at least three independent design parameters, this linkage inherently has a great flexibility for application to a wide number of design situations. Linkages can be used efficiently to amplify motion and the output element can be a low inertia producing mass, removed from the working elements.

Some of the uses to which approximate straight line mechanisms have been put are:

Guidance of piston rods

Radio station indicators

Granite gang saws

Film projectors

Auger hole drillers

There is the possibility of future use as dwell mechanisms to replace cams, ratchets, or indexing gears to allow higher speed operation and to replace the slider crank for various applications.

Scope of This Study

Three types of mechanisms commonly known as classical straight line linkages, the Watt, Evans, and symmetrical mechanisms, were investigated in this study. At the present time synthesis of these linkages has generally been achieved by graphical trial and error means. Most previous investigators have been content to state general information and very little useful quantitative information has been published. Results which have been presented have been primarily concerned with giving the designer a method for finding the necessary link dimensions. Little work has been done to provide the designer with data on the length and accuracy of approximate straight line output.

An analytical solution of the symmetrical mechanism has been presented by Chebychev (2). His work allows a linkage to be constructed which will produce a selected length of straight line output with a specified deviation. But no control is extended to the size of the linkage and a very large linkage may be required to produce the length of straight line required. With the development of high speed computers, analytical methods have become much more feasible. In his Ph. D. thesis (1964), Delbert Tesar presented analytical solutions to the three types of classical mechanisms. These solutions were particularly adaptable to solution with the digital computer.

It is the primary purpose of this study to reduce the "trial and error" effort now required of the designer by providing information as to length and deviation of approximate straight line output of specific linkages. This should reduce the designer's problem by providing a systematic set of choices to optimize the resulting design. The derivation by Tesar was used as the basis of a computer analysis. The data derived from this analysis are presented in the form of charts so that large amounts of data may be condensed into a practical amount of space. With a minimum of calculations, one may determine from these charts the straight line qualities of a specific linkage.

A similar chart or nomograph was used by Wunderlich[3] for presentation of data for symmetrical linkages. These could be used to determine a parameter which was an indication of the accuracy of the straight line output for a specific linkage. This nomograph was too limited to be of much use to a designer.

CHAPTER II

PROCEDURES AND GENERAL NOTATION

In this chapter the general procedures used to obtain the data for this thesis and also the general notation used to represent the linkages studied will be outlined.

Procedures

The data obtained from this study were the parameters required to define a four-bar mechanism which will produce an approximate straight line output when certain design specifications are designated. In order to accomplish this task, computer programs were written for the Burroughs 220 digital computer. These programs, one for each type of mechanism to be investigated, were written to give, as output, many items of interest in this study. Some of these were: length of approximate straight line output, crank rotation angles, link lengths, initial crank angles, ratio of longest to shortest links, and coupler point position. From this output, two items were selected to be presented in this thesis, approximate length of straight line output and ratio of longest to shortest link. Charts were drawn on which contour lines represent the locus of those linkages which have equal length of straight line output and the chart coordinates are the design parameters. These charts condense many hundreds of pages of computer output into a few pages so that the results may be reviewed and conclusions be reached.

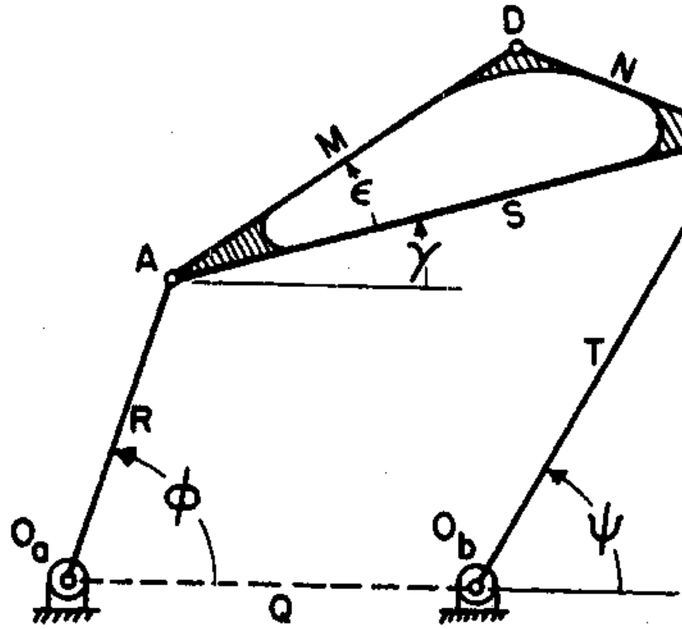


Figure 1. General Four-Bar Linkage

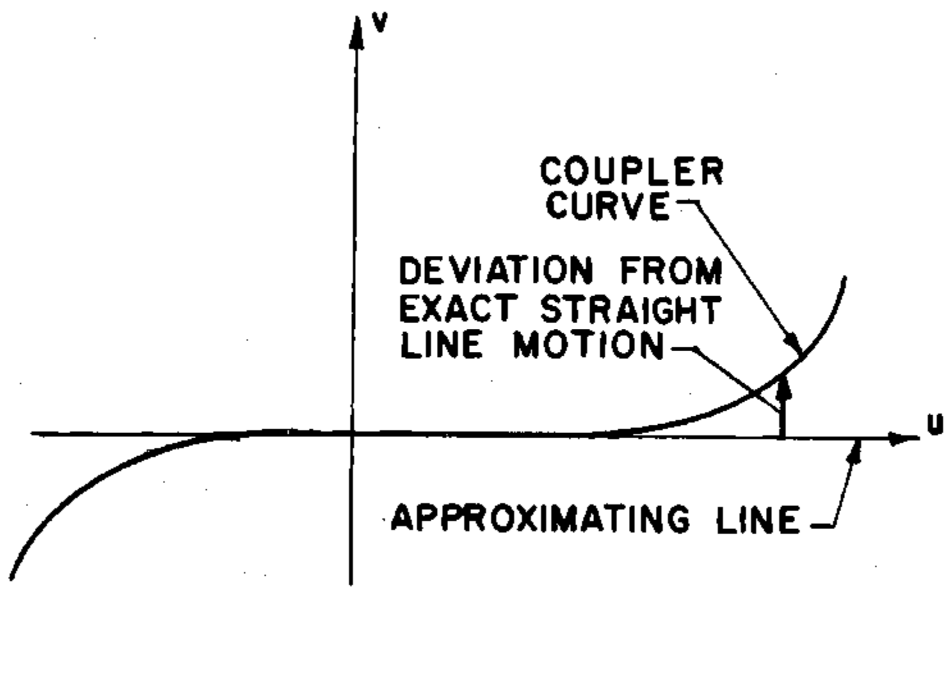


Figure 2. Deviation Curve

General Notation

In order that the reader might more easily follow the presentation of the research, an explanation of the terms and symbols to be used will be given here.

Basic Terms

Referring to Figure 1, the configuration of a general four-bar linkage, it may be noted that the two fixed pivots are designated O_a and O_b and the respective ends of the cranks as A and B. The point D is the coupler point. The length of the fixed link O_aO_b is designated Q, the length of the cranks $O_aA = R$ and $O_bB = T$. The length of the coupler AB = S. The line AD is length M and BD = N. The angle of rotation of crank R is defined by ϕ and the angle of rotation of the crank T by ψ. The angle γ is the transmission angle of A and the angle ε is the angle locating the coupler point with respect to the coupler center line. This notation was used throughout this work.

Some terms used in describing the properties or output of linkages are:

(a) Approximate Straight Line Output--the length of the coupler point path when it approximates a straight line within a certain specified accuracy (Figure 2).

(b) Ratio of Longest to Shortest Link--the ratio of the length of the longest link in the linkage to the length of the shortest link in the linkage. The link of maximum length is the largest value of Q, R, S, T, or $\frac{M+N}{2}$. The link of minimum length is the smallest of Q, R, S, T, or M+N. The notations of $\frac{M+N}{2}$ and M+N are used to represent one link when determining the maximum or minimum link length. This convention is used because when either M or N is very small it merely means that the coupler point is close to the coupler center line and is not detrimental to the operation of the linkage and since M+N will always be at least as large as S, it will never be the shortest link. But the maximum value of M and N must be considered because there is no limit on how far away from the coupler center line the coupler point can be. Therefore the values of M and N must be considered when selecting the largest link. Here again the value of M and N are not as important to the operation of the linkage as are the values of the other links because the coupler point is in a fixed position with respect to the coupler link.

Coordinate Systems

A uniform set of coordinate systems will be used throughout this work. The coordinates of the pin joints, the fixed pivots, and the output point in their initial positions are derived analytically. The location of the coordinate system is dictated by the requirements of the design equations. The set of coordinate systems is shown in Figure 3. The supporting theory allows the calculation of the coordinates in the U, V system of the following essential points necessary to define the linkage:

$O_a(U_1, V_1)$ $A\ (UP_1, VP_1)$	The coordinates of the endpoints of the input crank.
$O_b\ (U_2, V_2)$ $B\ (UP_2, VP_2)$	The coordinates of the endpoints of the output crank.
D (UPI, VPI)	The coordinates of the output point in the coupler plane.

COORDINATES:

A	UP_1 , VP_1	O_a	U_1 , V_1
B	UP_2 , VP_2	O_b	U_2 , V_2
D	UPI , VPI		

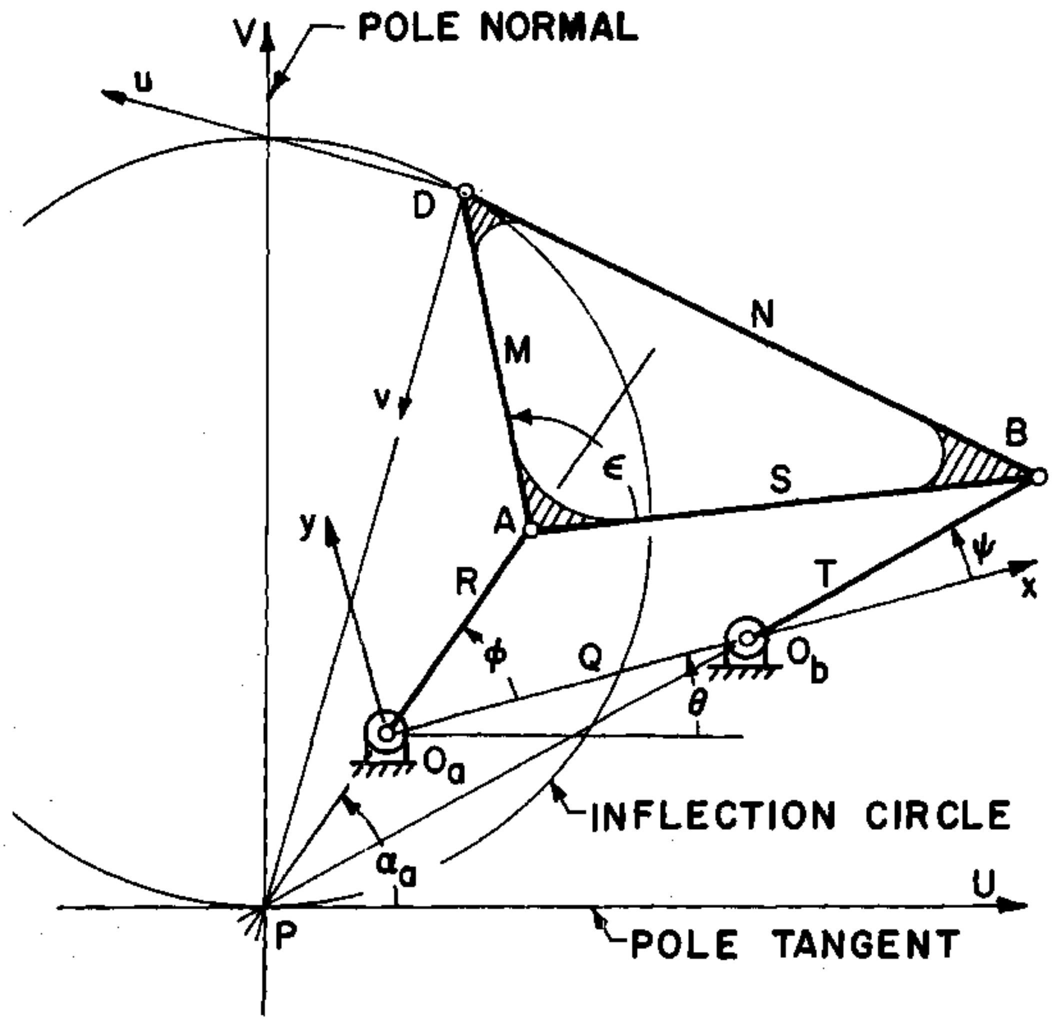


Figure 3. General Set of Coordinate Systems

The u, v coordinate system is used to represent the travel in the u direction of the output point D and the deviation in the v direction of the coupler curve from the true straight line. Its origin is located at the initial position of the point D.

The x, y coordinate system is used when calculating the linkage dimensions in the initial position. This coordinate system is attached to the fixed link with the origin at one of the pivots and the x-axis directed through the other fixed pivot.

CHAPTER III

WATT MECHANISM

Basic Motion of the Watt Mechanism

The limiscoid or Watt motion was the basis of perhaps the first mechanism actually used to produce approximate straight line motion by means of a pinned linkage. In the design or initial position, the control cranks are parallel so that their intersection which is the instant center of velocity for the coupler link relative to the fixed link is at infinity. Consequently, the coupler link in this position has pure translatory motion and all points in the coupler plane momentarily describe path elements that have zero curvature. One point exists on the coupler link which will continue to move in pure translation for continued rotation of the cranks. Hence this point will produce straight line motion.

The location of this coupler point for the Watt mechanism may be found as follows:[4]

Figures 4 and 5 show a mechanism O_aABO_b with $\overline{O_aO_b}$ defining the fixed link. The length $\overline{O_aA}$ is denoted R (the crank) and the length $\overline{O_bB}$ is denoted T (the second crank or follower). For a small rotation $d\phi$, the link AB takes a new position A'B' by virtue of pure translational motion. The corresponding small rotation of O_bB is denoted by $d\psi$. Therefore

$$R \cdot d\phi = T \cdot d\psi$$

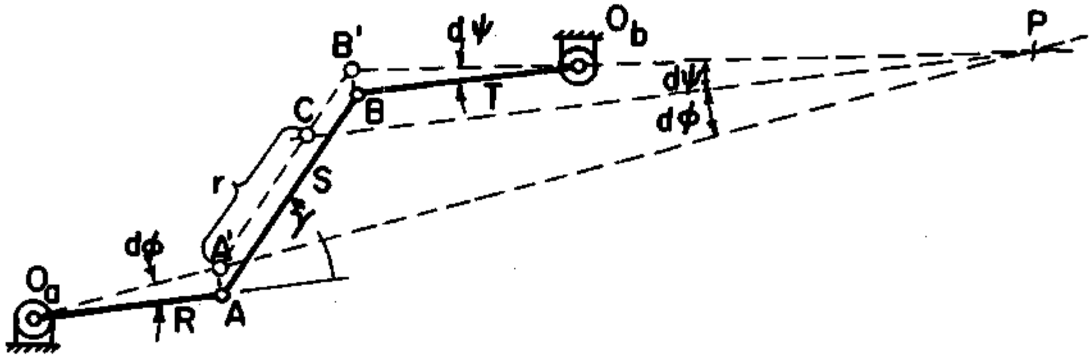


Figure 4. Watt Mechanism (Crossed Type)

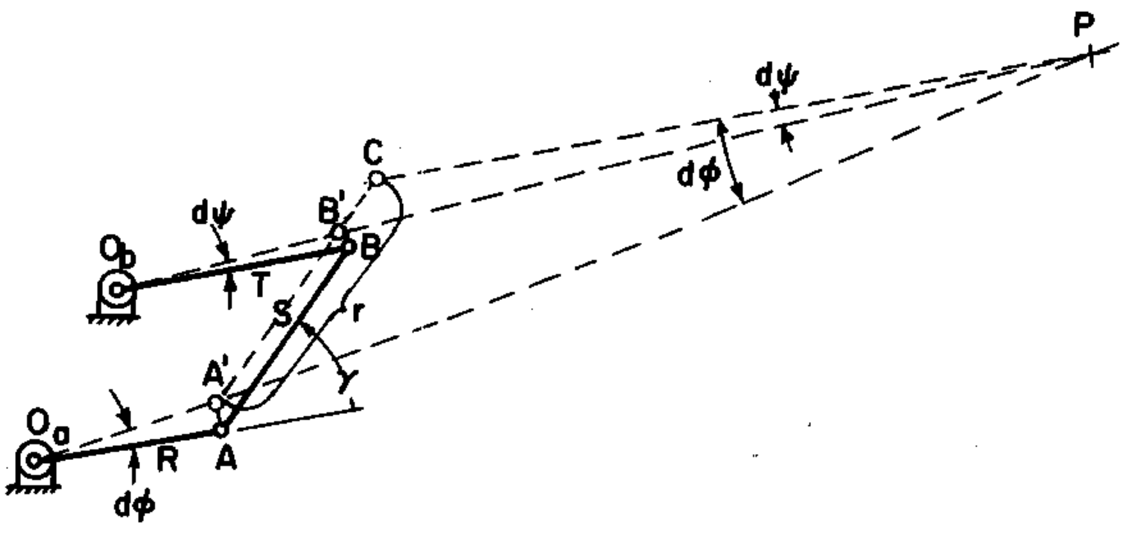


Figure 5. Watt Mechanism (Open Type)

The point C on A'B' that will continue to move in a straight line is located by drawing through point P a line $\overline{PC}$ // $\overline{O_aA}$ // $\overline{O_bB}$ where P is the intersection of lines $\overline{O_aA'}$ and $\overline{O_bB'}$. The point C divides the link A'B' into two parts, r and (S-r), where S is the length of link AB. The ratio of these two parts will be determined for use in locating point C.

By the law of sines

$$\frac{r}{\sin d\phi} = \frac{\overline{PC}}{\sin(\gamma-d\phi)}$$

$$\frac{\nu(s-r)}{\sin d\psi} = \frac{\overline{PC}}{\sin(\pi-\gamma-d\psi)}$$

where ν = +1 for crossed type linkages and ν = -1 for open type linkages. As classified here, an open linkage is one in which the line of the coupler link does not intersect the fixed link and a crossed linkage is one in which the line of the coupler link does intersect the fixed link. Since $d\phi$ and $d\psi$ are small, we can replace their sines with the value of the angle. Also the value of $d\phi$ and $d\psi$ may be neglected in comparison with π and γ. Thus

$$\frac{r}{d\phi} = \frac{\overline{PC}}{\sin\gamma}$$

$$\frac{\nu(S-r)}{d\psi} = \frac{\overline{PC}}{\sin\gamma}$$

and therefore

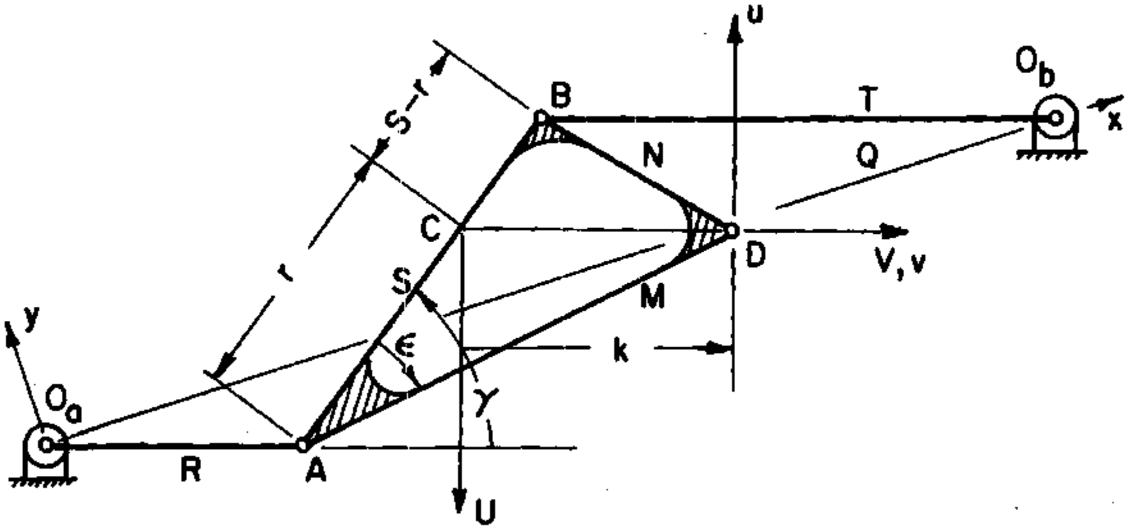


Figure 6. Watt Mechanism (Crossed Type).

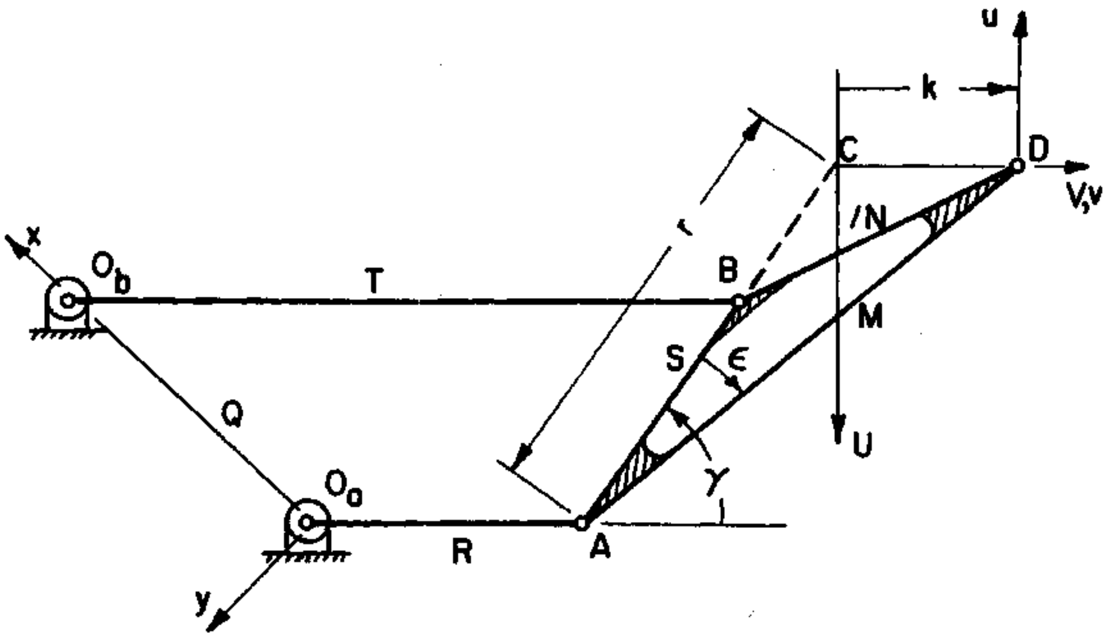


Figure 7. Watt Mechanism (Open Type).

$$\frac{r}{d\phi} = \frac{\nu(S-r)}{d\psi}$$

Since $R \cdot d\phi = T \cdot d\psi$

$$\frac{R}{T} = \frac{d\psi}{d\phi}$$

$$\frac{T}{R} = \frac{1}{\nu} \; \frac{r}{(S-r)} \tag{3-1}$$

Letting $\eta = \frac{1}{\nu}$, then

$$r = \frac{TS}{T+\eta R} \; . \tag{3-2}$$

Using this expression, the coupler output point is easily located. Other points not lying on the coupler center line, point D (Figures 6 and 7), also may produce satisfactory approximate straight line motion. These points are those on a line parallel to links $\overline{0_aA}$ and $\overline{0_bB}$ in the initial position and through point C. The parameter $k = \overline{CD}$ will be used to specify the location of D along this line.

Referring again to Figures 6 and 7 it can be noted that the coordinate system fixed to the moving coupler plane is denoted by the variables U and V. The coordinates of the pin joints of the mechanism were derived by D. Tesar[1] and reproduced in Appendix A. Expressions for the link dimensions and angles are also included in this same section.

Method of Compiling and Presenting Results

A study of the various Watt mechanisms was accomplished using computer programs which were written to yield a number of characteristics including the length of approximate straight line output and ratio of longest to shortest link. In order to obtain meaningful data, it was necessary to present each linkage scaled to a size representative with all others. This was accomplished by incorporating a "unit length" or unit number (UN) into the computer program. This unit number is defined as:

$$UN = \frac{Q + R + S + T + \frac{M + N}{2}}{5}$$

where the symbols denote link lengths as explained in Chapter II. Each linkage investigated was divided by such a number to give a basis for size comparison of linkages.

The straight line output was plotted within deviation ranges of 0.01, 0.001, and 0.0001 unit. That is, the path of the coupler point was plotted and the length measured to the point where the perpendicular deviation from the exact straight line just exceeded the specified value of the deviation. Then the sum of the lengths plotted on each side of the design position gave the length of approximate straight line output.

Assuming the coupler link to equal one unit, there are five independent design parameters for the general Watt mechanism: (a) the length of the input crank R, (b) the length of the output crank T, (c) the angle of the coupler with respect to the cranks in the design position γ, (d) the distance the coupler point is located from the coupler center line k, and (f) whether an open or crossed type linkage is to be considered. In order that the number of possible linkages be reduced from $2x\infty^4$ to a number that may be investigated practically, some design parameters had to be eliminated or their ranges limited. Only the special case in which the coupler point is located on the center line of the coupler was considered in this investigation. A short analysis of the effects of this limitation on the generality of the problem is discussed later in this chapter. Also, to keep the linkages within practical ratios of longest to shortest link, both input and output crank lengths were limited to a range from 0.4 to 3.0 units. For practical configurations, the angle of the coupler with respect to the cranks in the initial position was considered for values of 30°, 45°, 60°, 75°, and 90°. This limited the analysis to those mechanisms with high probability of practical usefulness.

The results of the computer program were plotted on charts W1 through W7 in Appendix B. The orthogonal coordinates of these charts are the crank lengths R and T. Separate charts are presented for crossed and open type linkages and one chart of each of these is presented for each value of the coupler angle γ. The value of the maximum deviation is also noted on each chart. Because of symmetry only the half of the charts for R less than T are included. The contour lines on the charts represent the length of approximate straight line output of given deviation for the linkage which has the specifications given by the coordinates of the point on the contour line. Therefore from these charts the approximate straight line output of a particular mechanism may be found.

Results Obtained from Charts

A review of the charts W1 through W7 reveals that the greater the initial angle of the coupler with the cranks from 30° to 75°, the greater the length of the straight line output for a linkage of given dimensions. This results from the fact that for a given angle of rotation of crank R, the coupler point C will move a greater distance when the angle γ is near perpendicularity. Also the nearer the coupler angle is to 90°, the less the coupler plane rotates for any given rotation of the crank R. In the range of γ from 75° to 90° the typical approximate straight line output drops off slightly. In all cases the linkages of best output tend to have values of R and T at least twice the length of the coupler and with R from 2/3 to 3/4 the length of T.

Another interesting observation is that in every case when the remaining design parameters are the same, the open type linkage will produce a longer straight line output than the crossed type. Yet the crossed type mechanism is more popular among designers at the present time.

Coupler Point Not Located on the Coupler Center Line

It was noted earlier in this chapter that the coupler point can be located a distance from the center line of the coupler. Because of the added design parameter which produces an impossibly large number of mechanisms to be studied, a complete study of the effects of this design parameter k could not be undertaken here. But, for a general comprehension of the effects a short survey was included with some typical results presented in Table 1. From these few cases, it is noted that for open type mechanisms, the length of approximate straight line output for a given mechanism will increase for negative values of k when the angle γ is less than 60°. For the angle γ greater than 60°, the approximate straight line output increases for positive values of k and decreases for negative values of k. Some exceptions are noted in which the output decreases for all values of $k \neq 0$. Also the output tends to decrease in all cases as k becomes large.

For crossed type mechanisms, the approximate straight line output tends to increase for both positive and negative values of k, but the increase is limited to small values of k. This limiting effect on the size of the absolute value of k for an increased length of output becomes more pronounced as the value of the angle γ increases.

Hence it can be seen that more flexibility in design and use of the Watt mechanism may be obtained if consideration is given to locating the coupler point off the coupler center line. Considering the trends mentioned above, one should be able to predict the results for mechanisms having the coupler output point not located on the coupler center line.

Table 1. Length of Straight Line Output for General Watt Mechanism

OPEN TYPE MECHANISMS $\eta = -1$

$\gamma = 30°$ R = 1.8

T = 2.0	0.309	0.318	0.314	0.310	0.334	0.345	0.356	Length of
T = 2.8	0.471	0.494	0.522	0.533	0.554	0.582	0.603	straight line
k =	1.5	1.0	0.5	0.0	-0.5	-1.0	-1.5	output

$\gamma = 45°$ R = 1.0

T = 2.0	0.494	0.526	0.560	0.596	0.617	0.624	0.623	Length of straight
T = 2.8	0.490	0.511	0.540	0.590	0.616	0.627	0.618	line output
k =	1.5	1.0	0.5	0.0	-0.5	-1.0	-1.5	

$\gamma = 75°$ R = 1.0

T = 1.4	1.010	0.964	0.926	0.910	0.850	0.815	0.773	Length of
T = 2.8	0.672	0.736	0.836	1.019	0.952	0.762	0.658	straight line
k =	1.5	1.0	0.5	0.0	-0.5	-1.0	-1.5	output

$\gamma = 90°$ R = 1.0

T = 2.0	1.443	0.996	0.915	0.863	0.814	0.767	0.712	Length of
T = 2.8	0.825	0.982	1.260	0.880	0.767	0.703	0.648	straight line
k =	1.5	1.0	0.5	0.0	-0.5	-1.0	-1.5	output

CROSSED TYPE MECHANISMS $\eta = +1$

$\gamma = 30°$ R = 1.8

T = 2.6	0.461	0.495	0.484	0.395	0.434	0.434	0.409	Length of
T = 2.8	0.508	0.469	0.482	0.418	0.414	0.423	0.396	straight line
k =	1.5	1.0	0.5	0.0	-0.5	-1.0	-1.5	output

$\gamma = 45°$ R = 1.6

T = 2.6	0.576	0.693	0.710	0.505	0.576	0.542	0.483	Length of
T = 2.8	0.524	0.592	0.703	0.482	0.565	0.529	0.469	straight line
k =	1.5	1.0	0.5	0.0	-0.5	-1.0	-1.5	output

$\gamma = 75°$ R = 1.8

T = 2.0	0.596	0.677	1.030	0.730	0.984	0.682	0.578	Length of
T = 2.8	0.560	0.625	0.962	0.702	0.851	0.638	0.550	straight line
k =	1.5	1.0	0.5	0.0	-0.5	-1.0	-1.5	output

$\gamma = 90°$ R = 1.8

T = 2.0	0.576	0.661	0.825	1.043	0.918	0.984	0.831	0.661	0.560	Length of
T = 2.8	0.548	0.619	0.760	1.090	0.894	0.932	0.807	0.624	0.535	straight
k =	1.5	1.0	0.5	0.25	0.00	-0.25	-0.5	-1.0	-1.5	line out-put

CHAPTER IV

EVANS MECHANISM

Basic Motion of Evans Mechanism

The general form of the Evans mechanism is based on the Cardanic circles which produce a special form of cycloidal motion. This cycloidal coplanar motion is generated by a circle rolling on the inner surface of a fixed circle of twice the diameter of the moving circle (Figure 8). All points rigidly attached to the smaller circle (such as A) describe elliptical point paths. Those points on the surface of the smaller circle (such as B and D) trace exact straight line point paths along diameters of the larger fixed circle.

If any two points (Example: points B and D in Figure 9) of a moving plane are momentarily tracing exact straight line point paths, the remaining points (Example: point A) of the moving plane trace elliptical coupler curves, segments of which may be approximated by a circular arc (Example: the arc described by the radius O_aA). If point B is guided by a slider and A is controlled by a rigid crank O_aA, the resulting slider crank mechanism will have an approximate straight line output at all points such as D. If the straight line point path of B is satisfactorily approximated by a circular arc of sufficiently large radius (O_bB) the resulting four-bar linkage is known as an Evans mechanism. The accuracy of the straight line output at D will depend upon the quality of the approximations at A and B.

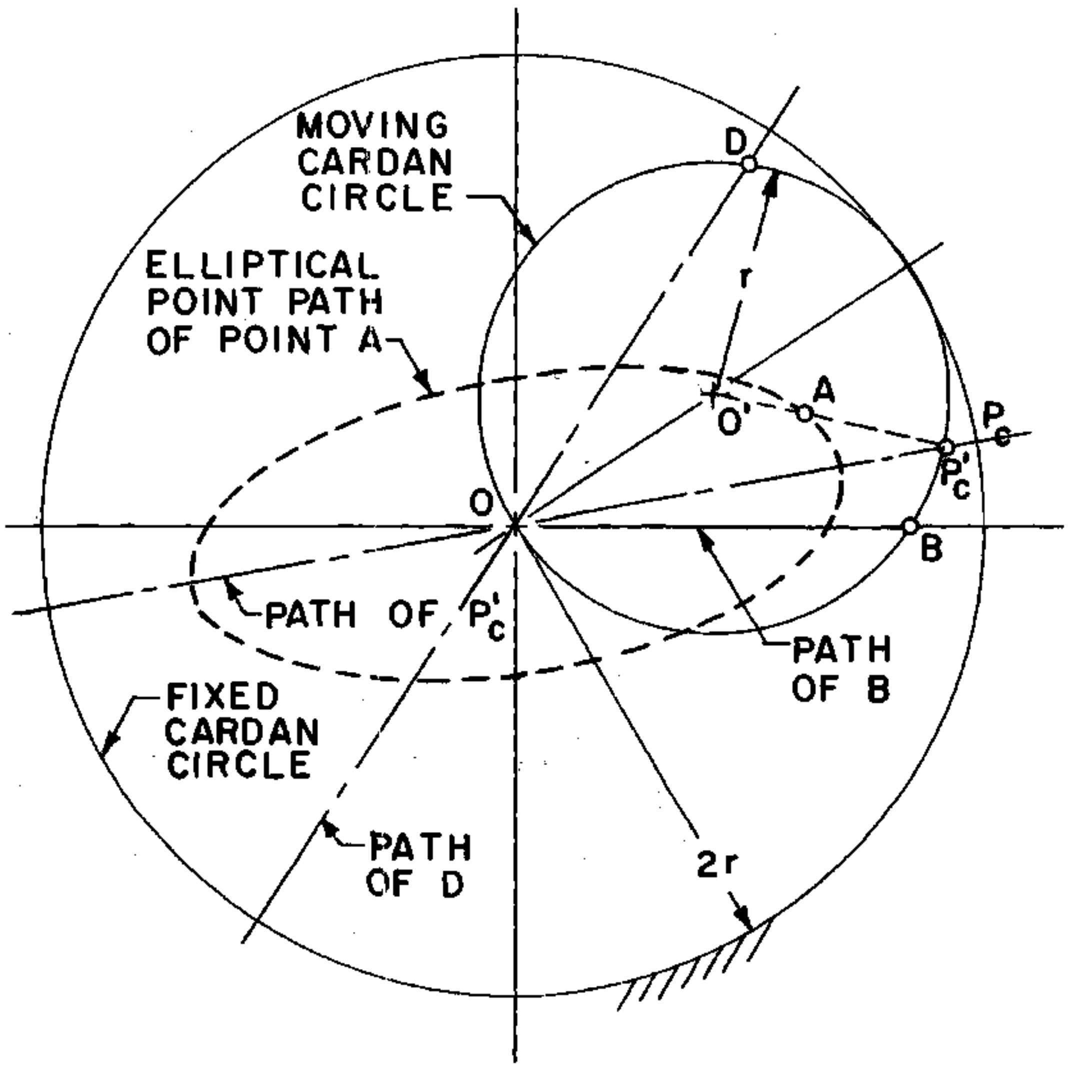


Figure 8. Cardan Motion.

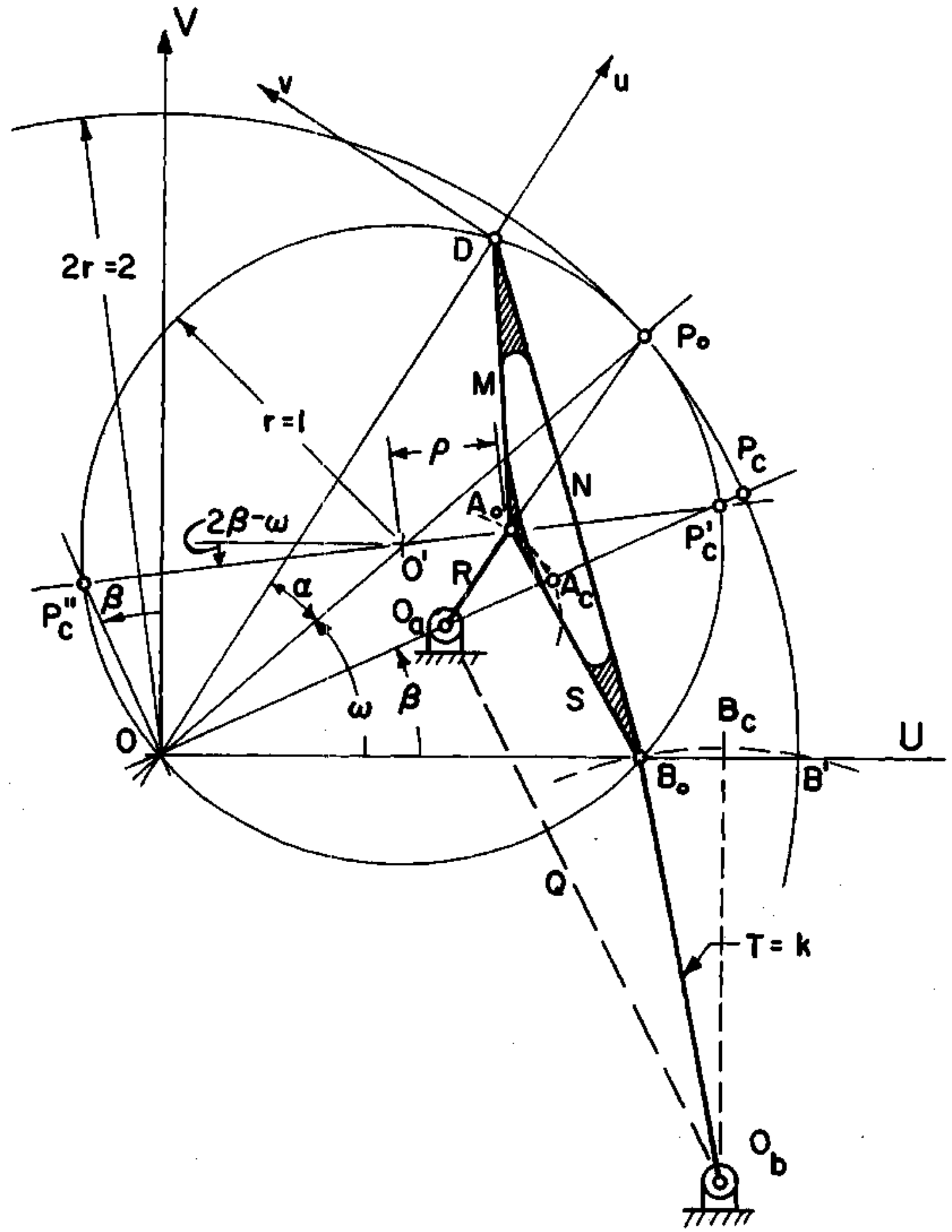


Figure 9. General Form of the Evans Mechanism.

Referring to Figure 9 it can be noted that again the fixed coordinate system is designated by U, V and the coordinate system moving with the coupler plane designated by u, v. Expressions for the positions of the pin joints and coupler points of the Evans mechanism in the U, V system were derived by D. Tesar[1] and are included in Appendix A. Expressions for link dimensions and angles are also given in terms of the design parameters in this same section.

Method of Compiling Data

Computer programs were written to analyze the characteristics of various Evans mechanisms. The most important property obtained was the length of the approximate straight line motion within specified deviations. Also important in this analysis is the ratio of the longest to shortest link. In order to compare the qualities of one mechanism with others it is necessary to reduce all mechanisms to the same relative size by the use of the unit number (UN). As before

$$UN = \frac{Q + R + S + T + \frac{M+N}{2}}{5}$$

The approximate straight line output was calculated for three deviation ranges: 0.05, 0.01, and 0.001 units.

For the Evans mechanism there are five basic design parameters. Referring to Figure 9, they are: (a) The length of the follower crank T denoted by k, (b) the angle β locating the position of the fixed point O_a, (c) the distance ρ locating the pin joint A on link R, (d) the angle

ω locating the initial position of the linkage, and (f) the angle α locating the output coupler point D on the smaller Cardan circle. Since each of these parameters has a theoretically infinite range, there are ∞^5 possible mechanisms. Of course it was necessary to reduce the number of mechanisms to be investigated. First the value of k was established at a fixed value of ± 2 units. The value of rho (ρ) was limited to the range between -1.0 unit and +1.0 until the angles beta (β) and omega (ω) were limited to from 0° to 90° and the angle alpha (α) was considered from -90° to +90°. These limits were established considering only the effects of the basic configuration of the design arrangements. Still the number of possible mechanisms remaining was too large to investigate thoroughly. To decide which area of these mechanisms to investigate further, an analysis of the entire range was made by calculating a relatively small number of linkages in each area of the entire range, and from the characteristics of these, deciding which areas to investigate more closely. The criteria for this decision was the relative length of approximate straight line output and the ratio of longest to shortest link. To simplify the problem of selecting the areas of mechanisms to be investigated, only areas where approximate straight line output was greater than 0.75 unit and ratio of longest to shortest link was less than 10 were calculated by the computer program.

The eight areas for which charts E1 through E14 are plotted were chosen as the areas of most useful data. And of these, the areas of best results were plotted for more than one deviation limit of approximate straight line output.

Note that the angle omega (ω) is not a parameter for the size of the linkage because it specifies the initial position of the linkage. Also the difference angle ($\omega-\beta$) is significant in predicting the accuracy of the resulting approximate straight line motion. If $\omega-\beta$ is small, the approximation will be quite accurate over a small range. If, however, $\omega-\beta$ is large, then the approximation will be poorer but the range will have been increased.

The Special Area for $\rho = 0$

One area of linkages which was investigated because it showed promise of containing useful linkages was the area for which $\rho = 0$. When results were obtained it was noticed that the parameter β seemed to have no effect on the outcome of the analysis. Further investigation of this fact showed that truly the parameter β had no effect and could be omitted from the necessary design parameters, a proof of this:

The location of the pin joint A_o from Equation (A-12) in Appendix A for $\rho = 0$ is (Refer to Figure 9)

$$VP_1 = \sin\omega \qquad (4\text{-}1)$$

$$UP_1 = \cos\omega$$

Note that these equations are now independent of β. The slope of line A_oP_o is

$$m = \frac{\sin\omega}{\cos\omega} = \tan\,\omega \qquad (4\text{-}2)$$

such that the equation of this line in the U, V system is

$$V = 2\sin\omega + \tan\omega\,(U - 2\cos\omega) \qquad (4\text{-}3)$$

The equation of line OP_c is

$$U = V\cot\beta \qquad (4\text{-}4)$$

Lines OP_c and A_oP_o intersect at the desired location of fixed pivot O_a which has the coordinates from Equation (A-16).

$$V_1 = \frac{2\sin\omega - 2m\cos\omega}{1 - m\cot\beta} \qquad (4\text{-}5)$$

$$U_1 = \frac{2\sin\omega - 2m\cos\omega}{\tan\beta - m}$$

When the expression for m is substituted into Equation (4-5) it produces the new expression for the coordinates of the pivot O_a

$$V_1 = 0 \qquad (4\text{-}6)$$

$$U_1 = 0$$

and this result is obviously independent of β. Therefore the pivot O_a is always located at the point O when $\rho = 0$. Since the location of the pivot O_a was the only point dependent upon the value of β for its location, β may be eliminated as a design parameter when $\rho = 0$. This result is useful because it eliminates one design parameter in an area of linkages with very good relative straight line outputs.

Presentation of Results

The final results of the investigation are presented in charts E1 through E14 in Appendix B. In charts E1 through E8 the orthogonal coordinates are ρ and α with the values of k, ω, and β given above each plot as a constant for that plot. On charts E9 through E14 the orthogonal coordinates are ω and α with values of k and ρ given above the charts. No value of β is necessary because the results are independent of β. The range of values of the ratio of the longest to shortest link is also noted on all charts. Because of the small size of the maximum ratio and also the small range of values, it was not necessary to plot values of the ratio corresponding to individual linkages in most cases. The contour lines indicate the length of approximate straight line output within the deviation specified. From any of the charts a point with a certain approximate straight line output may be selected. Then from the coordinates of this point and the constants on the chart, the linkage which will produce this motion can be constructed.

Conclusions from Results

More emphasis was placed on the area where $\rho = 0$ and charts including all three accuracy ranges were plotted. This was because this group of linkages not only produced relatively long straight line outputs within the largest deviation, but maintained above average output characteristics when smaller deviations were investigated. Also this is the area of investigation requiring only four design parameters, two of which, ρ and k, may be considered almost as constants. The value of ρ must be zero in order that β not be a factor and k is taken as ± 2 but the charts for these two values of k are very similar. Therefore one is at liberty to select from either chart depending upon the configuration needed for his specific case. From this set of charts it is also noted that when the angle ω-β, which is ω minus a constant, is large, the approximate straight line output is not very accurate but covers a large range or length. But when ω-β decreases, the accuracy of the approximate straight line output is better but the range is shorter. This can be noted because the maximum length of approximate straight line output at low accuracy, 0.05 unit, occurs at $\omega = 60°$. But maximum length of output occurs at $\omega = 45°$ for accuracy of 0.01 unit and at $\omega = 25°$ for accuracy of 0.001 unit. The general shape of the contour lines shows that the maximum points truly indicate the general trend of all points on the charts. These results could have been predicted from statements earlier in the chapter about the effects of the factor (ω-β).

Charts of other areas of good approximate straight line output are included but not necessarily for all investigated values of deviation. Those values of deviation not included were not suitable for use because of low values of straight line output.

On some charts it is noted that the length of approximate straight line output is below the value of 0.10 unit for some linkages. Such areas of the chart should not be considered useful because in this range of values, the output was obtained from an input crank rotation of only one degree in each direction from the design position or possibly in only one direction. Therefore this is not a true approximation of a straight line since the deviation limit was exceeded on the first or second step calculated by the computer program. But, this same linkage may have good characteristics when a greater deviation is allowed.

The areas on the charts which contain no values and also no grid lines represent areas where the approximate straight line output for 0.05 unit deviation was less than 0.75 units or the ratio of longest to shortest link was greater than 10. Such areas were not investigated because of the need to limit the number of linkages.

No one type of Evans mechanism will produce a significantly better straight line output than the other types. Since all values of ratio of links are in a usable range, for linkages having the same approximate straight line output, the final deciding factors must be configuration and angles of rotation and transmission. These properties are obtained most easily by plotting the linkage and determining their values graphically. Such a method is still somewhat trial and error but the charts provided will reduce the number of parameters to be determined by this procedure.

CHAPTER V

SYMMETRICAL MECHANISMS

Basic Motion of Symmetrical Mechanisms

For a study of general symmetrical linkages, no special design procedures need be used. Special cases of symmetrical linkages are the Roberts, Chebychev, and those resulting from curvature theory. All of these linkages may be specified (Figure 10) by the following parameters:

a: This parameter orients the fixed pivots about the center line of the linkages. If a is negative, the cranks are crossed.

c: This parameter is the altitude of the coupler triangle from the coupler link AB.

d: This parameter gives the spacing between the coupler and fixed link in the central position.

The fixed coordinate system is the x, y system and the coordinate system moving with coupler plane is the u, v system. Expressions for link dimensions in terms of parameters and the coordinate transforms were derived by D. Tesar[1] and are included in Appendix A.

Methods of Compiling and Presenting Results

A computer program was written to analyze the characteristics of the symmetrical linkages. The Two most important properties investigated were the length of approximate straight line output and the ratio of the longest to shortest link. Since no theory nor information was

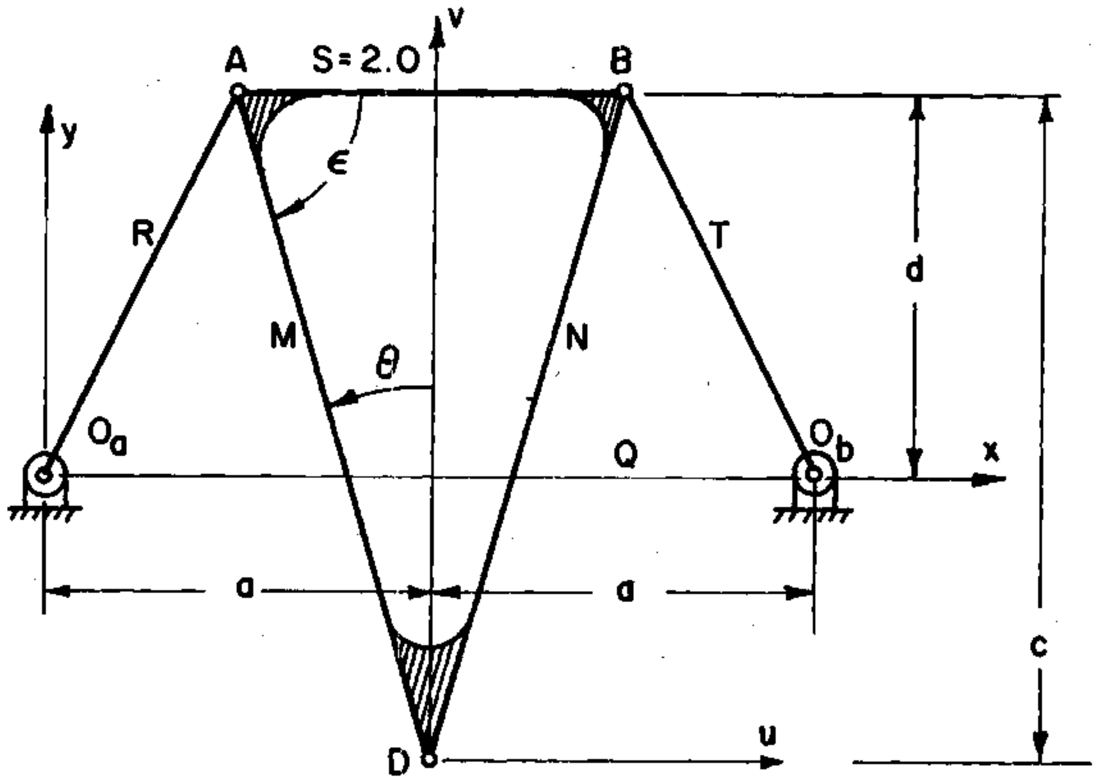


Figure 10. Symmetrical Mechanism.

available on what types of symmetrical linkages produce the most desirable characteristics, a survey of a large range of the parameters was made to find promising areas to be investigated more closely. As was the case with the other types of linkages, it was necessary to unitize the resulting linkages and outputs so that they might be effectively compared. Each was divided by a unit number (UN) defined by:

$$UN = \frac{Q + R + S + T + \frac{M + N}{2}}{5}$$

The areas determined as most desirable are those shown on Charts S1 through S12 included in Appendix B. Each of the areas is investigated for deviations of 0.05 and 0.01 unit.

The orthogonal coordinates are d and c. The parameter a is held constant for each chart but varied from chart to chart. The value of a ranges from -5.0 to +5.0 units. The values of c vary generally from -2.0 to 6.0 units, and values of d from 0.0 to 6.0 units. The contour lines on the charts represent the length of approximate straight line motion for the accuracy indicated. Also the ratio of longest to shortest link is included either as a note when the range is small or as superimposed contour lines in other cases. Additional properties of the symmetrical mechanism to be studied are easily formed graphically from the parameters which are the coordinates of the point on the chart which indicates the desired approximate straight line output.

Special Cases of Symmetrical Linkages

One special case of the symmetrical linkage is the Chebychev mechanism. The method of design for the symmetrical Chebychev mechanism as given by S. S. Block[2] involves the solution for the dimensions of the linkage from a given set of equations using two input parameters Θ_1, and Θ_4. The value of Θ_1 is the initial value of the angle between the vertical and the input crank and Θ_4 is the value of the same angle in the fourth and final position of the motion required to produce the approximate straight line. This set of equations was incorporated into a computer program and the results obtained for a large range of Θ_1 and Θ_4. These results were read out in the form of the design parameters a, c, and d used to specify a general symmetrical linkage. Where a linkage meeting the design requirement of the Chebychev linkage fell in a range of a, c, and d presented on one of the charts S1 through S12, the locus of the Chebychev linkage is denoted by a dashed line. On charts where no Chebychev linkages are designated, none were present. Only on charts S7, S8, S9, and S10 were Chebychev linkages found within the range of parameters a, c, and d. These were investigated.

Another special case of the symmetrical linkage is the linkage determined by curvature theory according to D. Tesar[1]. In this case the coupler output point coincides with the inflection pole and the cranks are specified equal. For these conditions there is only one input variable k which is defined as : $k = \frac{PO_a}{\text{Sin } \alpha_a}$ where PO_a is the directed distance from the fixed link pin joint to the inflection pole and α_a is the angle between the fixed link and the input crank in the initial position. A computer program was set up to determine linkages according to these specifications in terms of the design parameters a, c, and d. Where linkages produced from curvature theory were located in areas incorporated on charts S1 through S12 they were denoted by a point on the chart. Only one linkage was found that conformed to both curvature theory and also fell on the charts of general symmetrical linkages. This was on Chart S1 and S2. In all other cases the linkages conforming to curvature theory lay somewhere outside the range of a, c, and d which had been plotted.

CHAPTER VI

USE OF RESULTS FOR DESIGN PURPOSES

The charts of the approximate straight line output for the Watt, Evans, and symmetrical mechanisms may be used for design purposes as well as for a general survey into the characteristics of the various linkages. In this chapter will be included a short discussion of how to use each set of charts for design purposes along with a few examples. As a preliminary thought it should be carefully noted that if a mechanism selected from a chart is increased or decreased proportionately in size, the straight-line accuracy, as well as its length, change in the same proportion. For instance, if the size is increased to double the length of the straight line, the maximum deviation will double as well.

The Watt Mechanism Charts

In order to select a mechanism from the charts for the Watt mechanism, charts W1 through W7, the following procedure should be used.

1. Decide upon the type of Watt mechanism desired, crossed or open.

2. Select the coupler direction angle gamma wanted.

3. Read the crank lengths T and R from the appropriate chart for the length of straight-line and accuracy needed. The coupler link S is one unit long. The length of Q, M, and N can now be determined from the following equations:

PARAMETERS

$\eta = -1 \quad \gamma = 75^\circ \quad R = 1.0 \quad T = 2.0$

DIMENSIONS

Q =	0.9062	S =	0.7443
R =	0.7443	M =	1.4886
T =	1.4886	ϵ =	0.0

RESULTS

D	0.0001	0.001	0.01
L	0.195	0.427	1.793

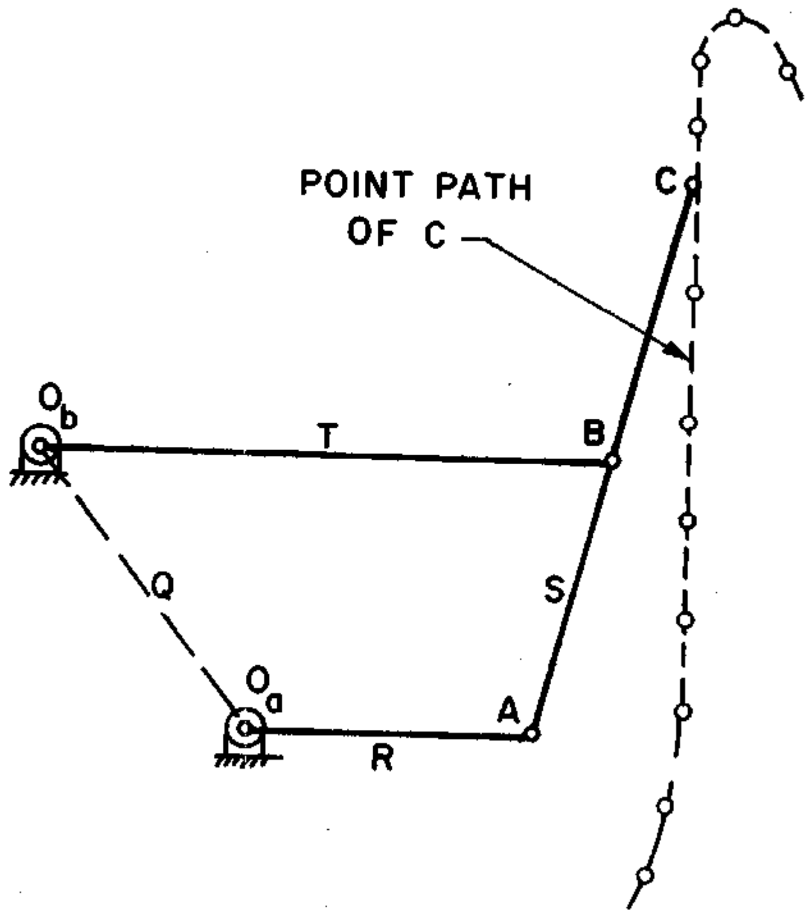


Figure 11. Example Watt Mechanism.

$$M = \frac{T_c S}{T_c + \eta R_c}$$

$$N = M - S$$

$$Q = S^2 + (T_c - R_c)^2 - 2S(T_c - R_c) \cos \gamma$$

Then the length of all links must be adjusted by the factor UN in the following manner

$$T_{actual} = (T_{chart}) / (UN)$$

The unit number is calculated as indicated earlier.

4. If the selected proportions prove undesirable, other combinations can be selected which have the same or very nearly the same straight-line length.

5. If none of these serve the purpose well enough, the linkage can be expanded or contracted as needed. If the mechanism is enlarged, for instance, the approximate straight-line will lengthen by the same proportional amount; but so will the deviation. A decrease in size will similarly reduce the length as well as the deviation from a straight line.

Example

An example will illustrate the use of the charts, the execution of the steps indicated above, and the application of the UN factor (Figure 11). Take any point on any chart, say chart W4, for which,

$$\gamma = 75^\circ$$

$$\eta = -1$$

$$D_c = 0.01$$

$$T_c = 2.00$$

$$R_c = 1.00$$

$$L_c = 1.79$$

$$S = 1.00$$

Then:

$$M = \frac{T_c S}{T_c + \eta R_c} = \frac{2.00 \times 1.00}{2.00 - 1 \times 1.00} = 2.00$$

$$N = M - S = 2.00 - 1.00 = 1.00$$

$$Q = \sqrt{S^2 + (T_c - R_c)^2 - 2S(T_c - R_c) \cos \gamma}$$

$$= \sqrt{1.00^2 + (2.00 - 1.00)^2 - 2 \times 1.00 \times (2.00 - 1.00)\ 0.259}$$

$$= \sqrt{1.48} = 1.21$$

$$UN = \frac{Q + R + S + T + \frac{M+N}{2}}{5}$$

$$= \frac{1.21 + 1.00 + 1.00 + 2.00 + \frac{2.00 + 1.00}{2}}{5}$$

$$= 1.34$$

Thus:

$$Q = Q_c/UN = \frac{1.21}{1.34} = 0.90$$

$$R = R_c/UN = \frac{1.00}{1.34} = 0.75$$

$$S = S_c/UN = \frac{1.00}{1.34} = 0.75$$

$$T = T_c/UN = \frac{2.00}{1.34} = 1.49$$

$$M = M_c/UN = \frac{2.00}{1.34} = 1.49$$

Therefore the mechanism which produces the length of straight line L_c = 1.79 unit within D_c = 0.01 unit deviation is:

$$Q_a = 0.90$$

$$R_a = 0.75$$

$$S_a = 0.75$$

$$T_a = 1.49$$

$$M_a = 1.49$$

$$\gamma_a = 75°$$

The Evans Mechanism Charts

In order to select a mechanism from the charts for the Evans mechanism, charts E1 through E14, the following procedure should be used.

1. Choose the parameter k to be +2.0 or -2.0.

2. Using the length of straight line and deviation required as a guide, select a point on one of the charts E1 through E14. The coordinates of the point and the constant values written above the chart will specify the linkage which produces the required straight line output.

3. Using the values of the parameters obtained in Step 2, calculate the linkage dimensions. For the general case where all five parameters are used, the method of obtaining the dimensions is to first calculate the coordinates of the pin joints from Equations (A-10) through (A-17) in Appendix A. From these coordinates the linkage dimensions can be calculated by the use of the general expression for the distance between two points in a plane:

$$\text{LENGTH} = \sqrt{(U_2-U_1)^2 + (V_2-V_1)^2}$$

A graphical layout of the mechanism will give the initial position. For the case when $\rho = 0$ and β is not included among the parameters, the coordinates in the U, V system are calculated from Equations (4-1), (4-6), (A-10), (A-11), and (A-17) in Chapter IV and Appendix A. Note that R = 1.0 and S = 1.0 for all linkages in this group. The length of all links must now be adjusted by the factor UN as was explained earlier for the Watt mechanism.

PARAMETERS

$k = -2.0 \quad \omega = 30° \quad \beta = 15° \quad \rho = 0.1 \quad \alpha = 60°$

DIMENSIONS

Q =	1.5964	S =	0.5974
R =	0.4888	M =	0.7104
T =	1.3091	N =	1.3062

RESULTS

D	0.001	0.01	0.05
L	0.109	1.364	1.975

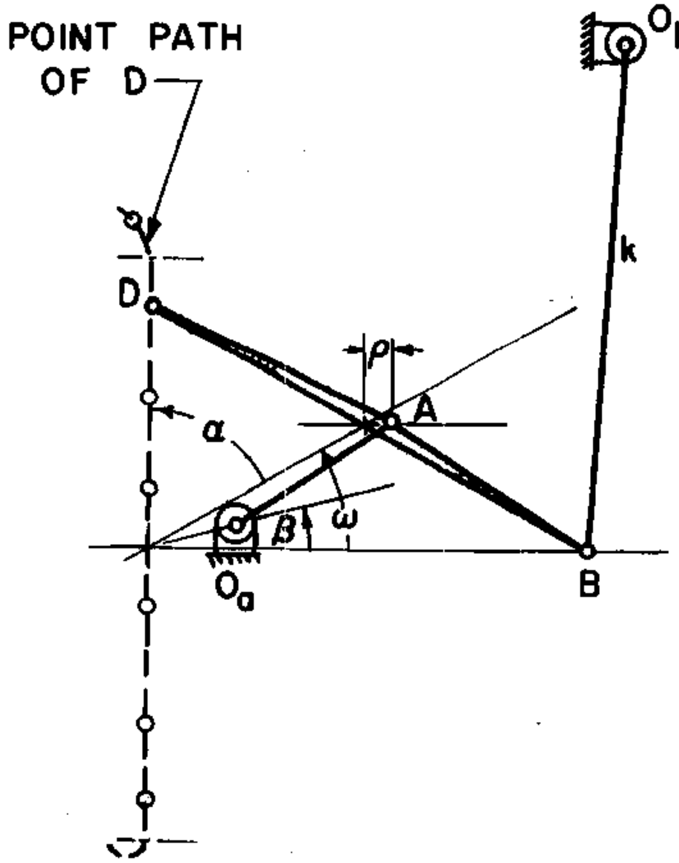


Figure 12. Example Evans Mechanism.

4. If the proportions of the calculated linkage prove undesirable, other combinations of the parameters may be selected from the same or other charts which include mechanisms of the same or very nearly the same straight line length.

5. If none of these serve the purpose well enough, the linkage may be expanded or contracted as needed.

Example

An example will illustrate the use of the charts and execution of the steps indicated above (Figure 12). Select any point on any chart, say,

$$K = -2.0$$

$$\omega = 30^{\circ}$$

$$\beta = 15^{\circ}$$

$$\rho = 0.1$$

$$\alpha = 60^{\circ}$$

$$L_c = 1.98$$

$$D_c = 0.05$$

Calculating the coordinates in the U, V system of the pin joints,

$$m = \frac{\text{Sin}\omega - \rho \sin(2\beta-\omega)}{\cos\omega - \rho \cos(2\beta-\omega)} \tag{A-13}$$

$$= \frac{\sin 30 - (0.1) \sin 2(15)-30}{\text{cis } 39 - (0.1) \cos 2(15)-30} = 0.653$$

$$V_1 = \frac{2 \sin\omega - 2m \cos\omega}{1 - m \cot \beta} \tag{A-15}$$

$$= \frac{2 \sin 30 - 2(0.653) \cos 30}{1-(0.653) \cot 15} = 0.091$$

$$U_1 = V_1 \cot\beta = (0.091) \cot 15 = 0.33$$

$$V_2 = -k = + 2.0 \qquad \text{(A-10)}$$

$$U_2 = 1 + \cos\omega = 1 + \cos 30 = 1.866$$

$$UP_1 = \sin\omega + \rho \sin (2\beta - \omega) \qquad \text{(A-12)}$$

$$= \sin 30 + (0.1) \sin[2(15)-30] = 0.500$$

$$VP_1 = \cos\omega + \rho\cos (2\beta-\omega)$$

$$= \cos 30 + (0.1) \cos[2(15-30] = 0.966$$

$$UP_2 = 2 \cos\omega = 2 \cos 30 = 1.732 \qquad \text{(A-11)}$$

$$VP_2 = 0$$

$$UPI = 2 \cos \alpha \cos (\alpha+\omega) \qquad \text{(A-17)}$$

$$= 2 \cos (60) \cos (60+30) = 0.0$$

$$VPI = 2 \cos \alpha \sin (\alpha+\omega)$$

$$= 2 \cos (60) \sin (60+30) = 0.500$$

Then calculating the dimensions

$$Q_c = \sqrt{(U_2-U_1)^2 + (V_2-V_1)^2}$$

$$= \sqrt{(1.866-0.33)^2 + (2.00-0.091)^2} = 2.44$$

$$R_c = 0.74$$

$$S_c = 0.91$$

$$T_c = 2.00$$

$$M_c = 1.09$$

$$N_c = 2.00$$

Therefore

$$UN = \frac{Q+T+S+T+\frac{M+N}{2}}{5} = 1.53$$

And

$$Q = Q_c/UN = \frac{2.44}{1.53} = 1.60$$

$$R = R_c/UN = 0.49$$

$$S = S_c/UN = 0.60$$

$$T = T_c/UN = 1.31$$

$$M = M_c/UN = 0.71$$

$$N = N_c/UN = 0.31$$

These dimensions specify the mechanism which will produce the specified output, a length of straight line $L_c = 1.98$ within a deviation of $D_c = 0.05$.

The Symmetrical Mechanisms Charts

To use the charts for the symmetrical mechanisms, charts S1 through S-12, one should use the following procedure.

1. Decide whether a mechanism with the cranks crossed or uncrossed is desired, a will be plus or minus, respectively.

2. From one of the charts for the type of mechanism selected in Step 1, choose a point at which the straight line output satisfies the design requirements. From the coordinates of this point, c and d, and the constant value of a, the dimensions of the mechanism may be calculated. The equations (A-18) in Appendix A and S = 2.0 are used for this calculation. Then the length of all links must be adjusted by the factor UN as was described earlier.

3. If the calculated proportions of the linkages prove undesirable, other combinations of the parameters may be selected which have the same or very nearly the same straight line length.

4. If none of these serve the purpose well enough, the linkage may be expanded or contracted as needed.

Example

An example will illustrate the use of the charts and the execution of the steps indicated above (Figure 13). Choosing any point on any chart.

PARAMETERS

$a = -3.0 \quad c = 1.0 \quad d = 2.5$

DIMENSIONS

Q	=	1.5917	S	=	0.5306
R	=	1.2513	M	=	0.3752
T	=	1.2513	N	=	0.3752

RESULTS

D	0.001	0.01	0.05
L	0.768	0.893	1.130

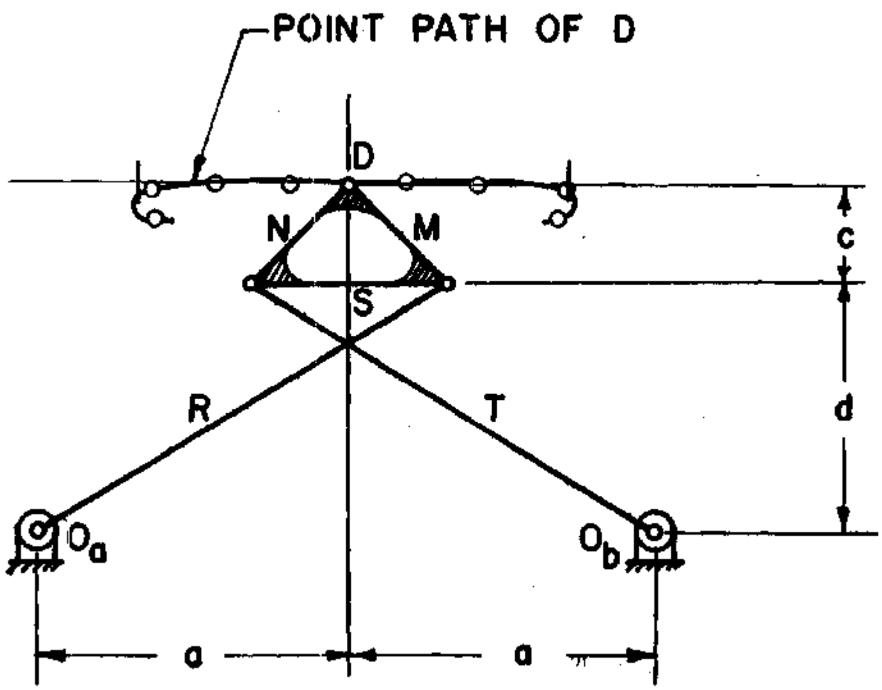


Figure 13. Example Symmetrical Mechanism.

$$a = -3.0$$

$$c = 1.0$$

$$d = 2.5$$

$$L_c = 1.13$$

$$D_c = 0.05$$

Since a = -3.0, this is a mechanism with crossed cranks. Calculating the dimensions from equation (A-18)

$$S_c = 2.0$$

$$Q_c = |2a| = |2 \times -3.0| = 6.0$$

$$R_c = T_c = [d^2+(1-a)^2]^{1/2}$$

$$= [(2.5)^2+(1 +3.0)^2]^{1/2} = 4.71$$

$$M_c = N_c = [1 + c^2]^{1/2}$$

$$= 1 + [(1.0)^2]^{1/2} = 2.00$$

Hence:

$$Q = Q_c/UN = 6.0/3.77 = 1.59$$

$$R = R_c/UN = 1.25$$

$$S = S_c/UN = 0.53$$

$$T = T_c/UN = 1.25$$

$$M = N = M_c/UN = 0.38$$

This is a symmetrical mechanism which will produce the specified straight line L_c = 0.90 within the design deviation.

APPENDIX A

DERIVATIONS

The derivations by D. Tesar[1] of the coordinates for the pin joints and coupler points of the Watt, Evans, and symmetrical mechanisms are presented here as reference material.

Derivation of Coordinates of Watt Mechanism

The notation used here for the Watt mechanism is from Figures 6 and 7. Letting T be the longer crank, the output point C on the coupler link center line AB is located from pin joint A by the distance

$$r = \frac{T}{T+\eta R} S \tag{A-1}$$

where $\eta = +1$ represents the crossed type and $\eta = -1$ represents the open type. A more general form of the linkage is possible. Some points D, not lying on the coupler center line, may also produce satisfactory approximate straight line motion. These points are taken to be those lying on a line from the instant center through point C and specified by the parameter $k = CD$.

The location of the fixed pivot O in the U, V system is given by

$$U_1 = +r\mathrm{Sin}\gamma \tag{A-2}$$

$$V_1 = (R + r \cos \gamma)$$

where S = 1.0 and γ represents the angle formed by the coupler link AB with the cranks in the design position. The pin joint A is located by

$$UP_1 = U_1 \tag{A-3}$$

$$VP_1 = V_1+R$$

The coordinates of the fixed point O_b are

$$V_2 = V_1+R+\cos\gamma+\eta T \tag{A-4}$$

$$U_2 = U_1 - \mathrm{Sin}\gamma$$

and the coordinates of the pin joint B are

$$VP_2 = V_2 + \eta T \tag{A-5}$$

$$UP_2 = U_2$$

Finally, the location of the output point D is given by

$$VPI = k \tag{A-6}$$

$$UPI = 0$$

Such that

$$M = [(UP_1)^2 + (VP_1-k)^2]^{1/2} \quad \text{(A-7)}$$

$$N = [(UP_2)^2 + (VP_2-k)^2]^{1/2}$$

The length of the fixed link is

$$Q = [(U_2-U_1)^2 + (V_2-V_1)^2]^{1/2} \quad \text{(A-8)}$$

To obtain the value of the coupler angle ε, the law of cosines is used in the form

$$\varepsilon = \pm\cos^{-1}[\frac{M^2+1-N^2}{2M}] \quad \text{(A-9)}$$

where the negative sign is used when the product $(k)(\eta)$ is greater than zero.

Derivation of Coordinates of Evans Mechanisms

Figure 9, the General Form of the Evans Mechanism, shows the points used in the following derivation.

The initial position of the coupler (denoted by the subscript o) and the central position (denoted by the subscript c) have well-defined instant centers P_o and P_c'. Any crank which is to be added as a constraint to the motion such as O_aA must be on pole rays from these instant centers. The point P_c', rigidly attached to the outer surface of the smaller Cardan circle, moves along the diameter $2(OP_c)$, of the larger circle so that P_c' coincides with P_c in the central position. Hence

$P_c'OP_c''$ is the diameter of the smaller circle which coincides with the diameter $2(OP_c)$ of the larger circle in the central positions. The result is that the ellipse being traced by point A has $2(OP_c)$ as one of its axes of symmetry. If the crank O_aA is to approximate the elliptical path of A, its center of rotation must lie on OP_c.

The smaller Cardan circle has its center O' on a ray (defined by ω) from the origin of the U, V coordinate system (the center of the fixed Cardan circle) and its radius is r = 1. This circle must pass through the origin of the U, V system. The radius of the outer circle is $OP_c=2$. As shown in the figure, U_2 is the average of the U coordinates for B_o and B' so that arc B_oB_c best approximates the straight line which point B on the smaller Cardan circle describes. This gives the coordinates of O_b to be

$$U_2 = 1 + \cos\omega \qquad \text{(A-10)}$$

$$V_2 = -k$$

and the coordinates of B are

$$UP_2 = 2\cos\omega \qquad \text{(A-11)}$$

$$VP_2 = 0$$

where k will take sufficiently large positive or negative values to insure that arc B_oB_c satisfactorily approximates a straight line. In the initial position, the instant center P_o is located at

$$(2 \cos \omega, 2 \sin \omega)$$

Note that the slope of the diameter $P_c''P_c'$ is governed by the angle $(2\beta-\omega)$. Letting the directed distance of the pin joint A from the center O' of the smaller circle be ρ, the coordinates of A_o are

$$VP_1 = \sin\omega + \rho\sin(2\beta-\omega) \tag{A-12}$$

$$UP_1 = \cos\omega + \rho\cos(2\beta-\omega)$$

The slope of the line A_oP_o is

$$m = \frac{\sin\omega - \rho\sin(2\beta-\omega)}{\cos\omega - \rho\cos(2\beta-\omega)} \tag{A-13}$$

such that the equation of this line in the U, V system is

$$V = 2\sin\omega + m(U-2\cos\omega) \tag{A-14}$$

The equation of OP_c is

$$U = V\cot\beta \tag{A-15}$$

Lines OP_c and A_oP_o intersect at the desired fixed pivot O_a which has the coordinates

$$V_1 = \frac{2\ \sin\omega\ -\ 2m\ \cos\omega}{1\ -\ m\ \cot\ \beta} \qquad \text{(A-16)}$$

$$U_1 = \frac{2\ \sin\omega\ -\ 2m\ \cos\omega}{\tan\beta - m}$$

The location of D on the outer surface of the smaller Cardan circle is defined by the parameter α, such that

$$\text{VPI} = 2\ \cos\ (\alpha+\omega)\ \cos\ (\alpha) \qquad \text{(A-17)}$$

$$\text{VPI} = 2\ \sin\ (\alpha+\omega)\ \cos\ (\alpha)$$

The orientation of the u, v system which has its origin at D is given by the parameter α. Since the coordinates of all the pertinent points (A, B, O_a, O_b, D) are known, the linkage is completely determined. The determination of the usual link dimensions (Q, R, S, T, M, N) is not reviewed here since it is quite straightforward.

Derivation of Expressions for Link Dimensions of Symmetrical Mechanisms

In Figure 10 is shown the symmetrical linkage referred to in this section. All dimensions of the lqnks are related to the coupler link AB which is taken to have a fixed magnitude S = 2.0. The following formulas can be used to determine the link dimensions:

$$M = N = [1 + c^2]^{1/2}$$

$$\varepsilon = \tan^{-1}\ (-c)$$

$$Q = |2a|$$

$$R = T = [d^2 + (1-a)^2]^{1/2} \tag{A-18}$$

and the u, v system is given in terms of the x, y system as

$$\begin{aligned} u &= x - |a| \\ v &= y - d \pm c \end{aligned} \tag{A-19}$$

where the minus sign corresponds to the minus sign of the parameter a.

APPENDIX B

CHARTS OF COMPUTER OUTPUT

The results of the computer programs for the approximate straight line output of the Watt, Evans, and symmetrical mechanisms have been plotted on the charts included in Appendix B. The type of mechanism and other information necessary for the use of the charts is noted on each chart.

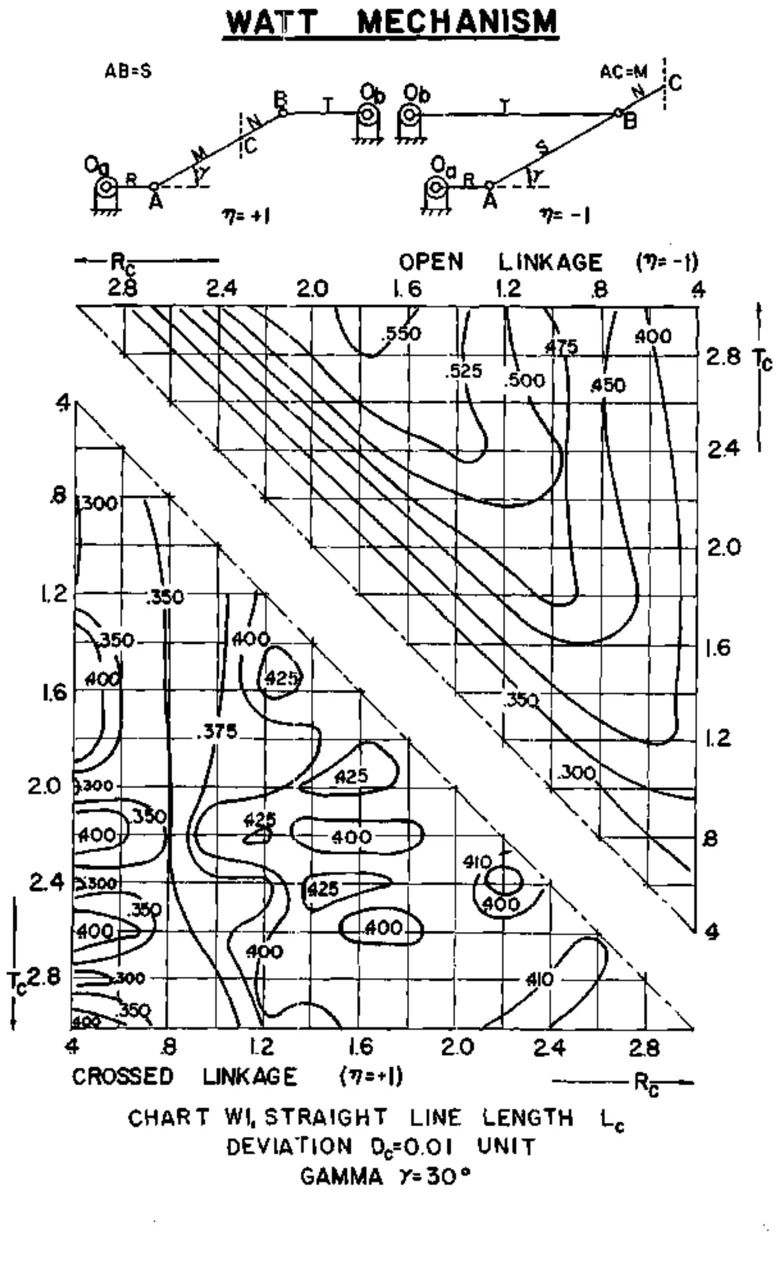


Figure 14. Watt Mechanism Chart W 1.

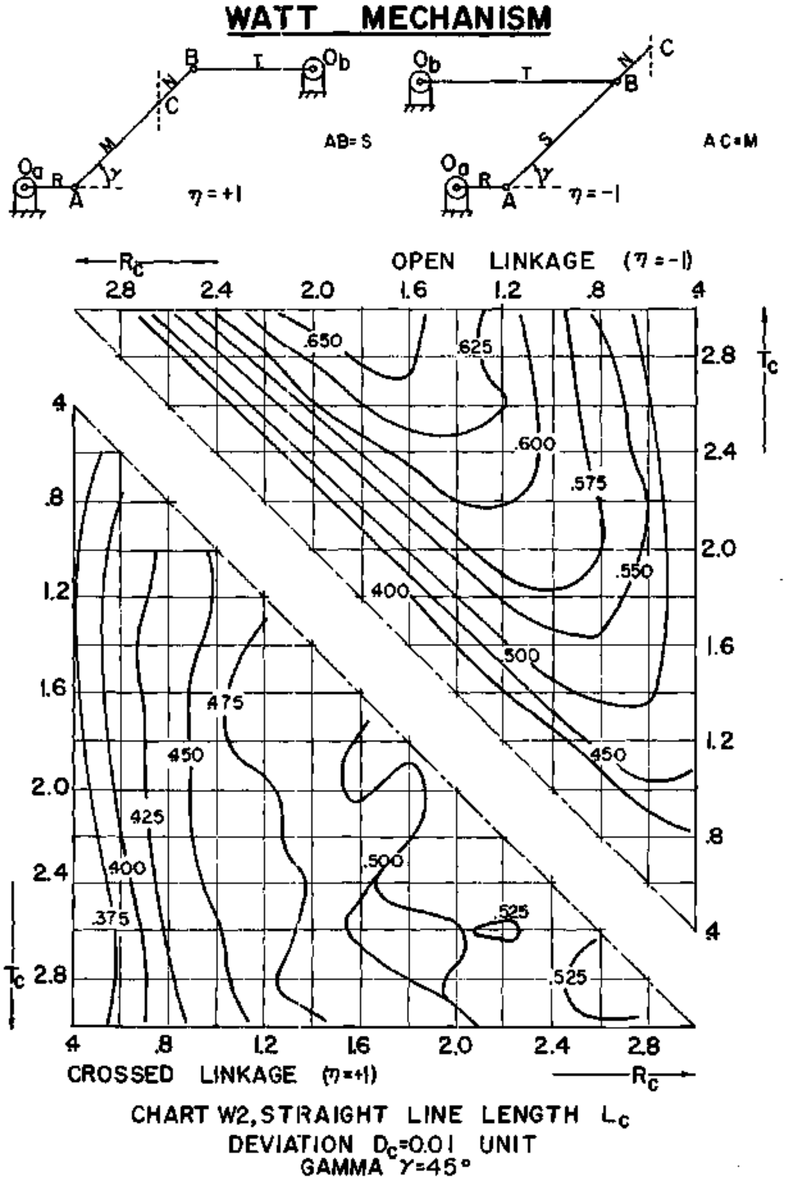


Figure 15. Watt Mechanism Chart W 2.

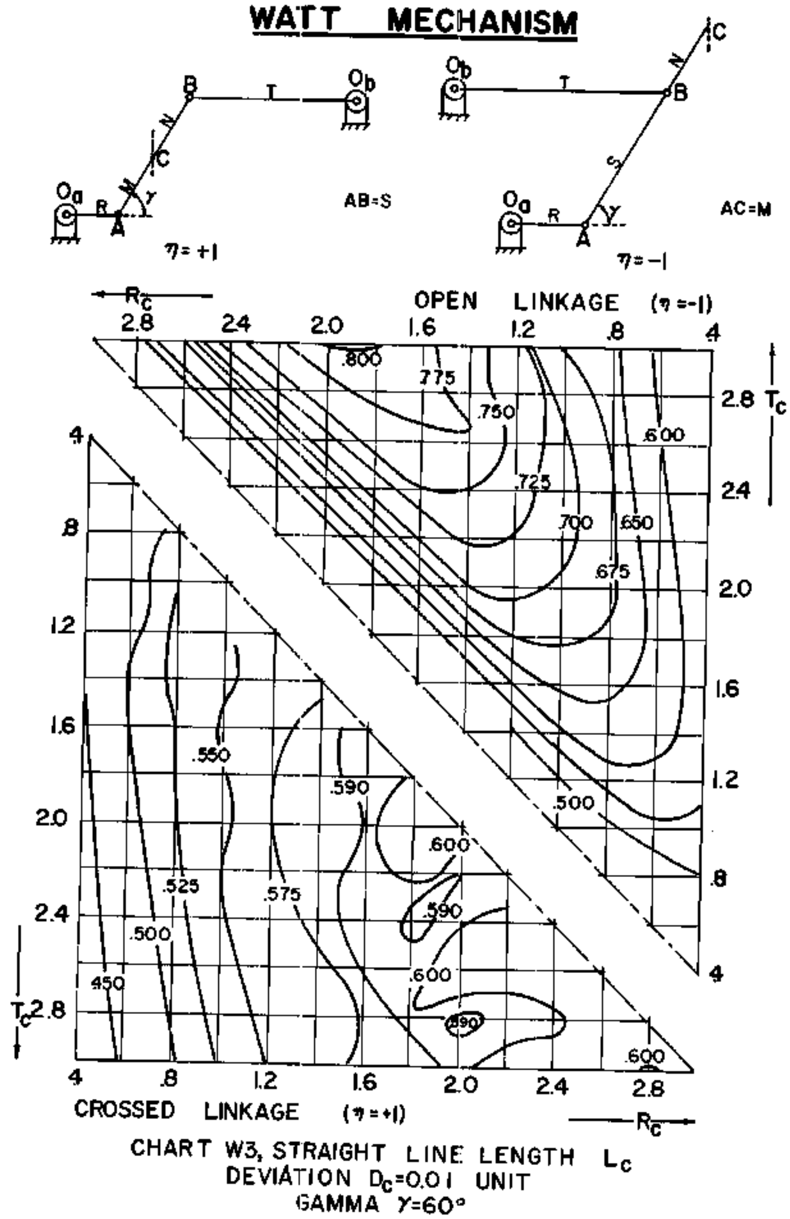


Figure 16. Watt Mechanism Chart W 3.

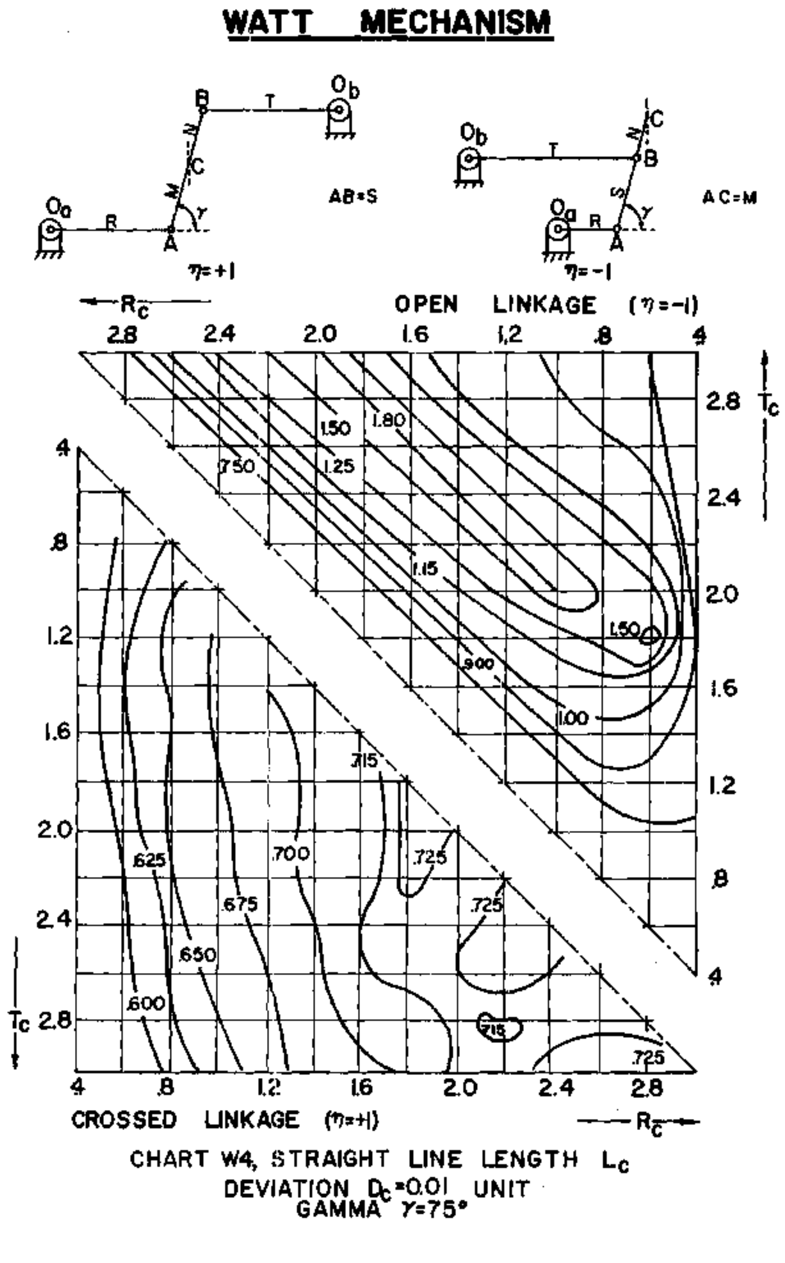


Figure 17. Watt Mechanism Chart W 4.

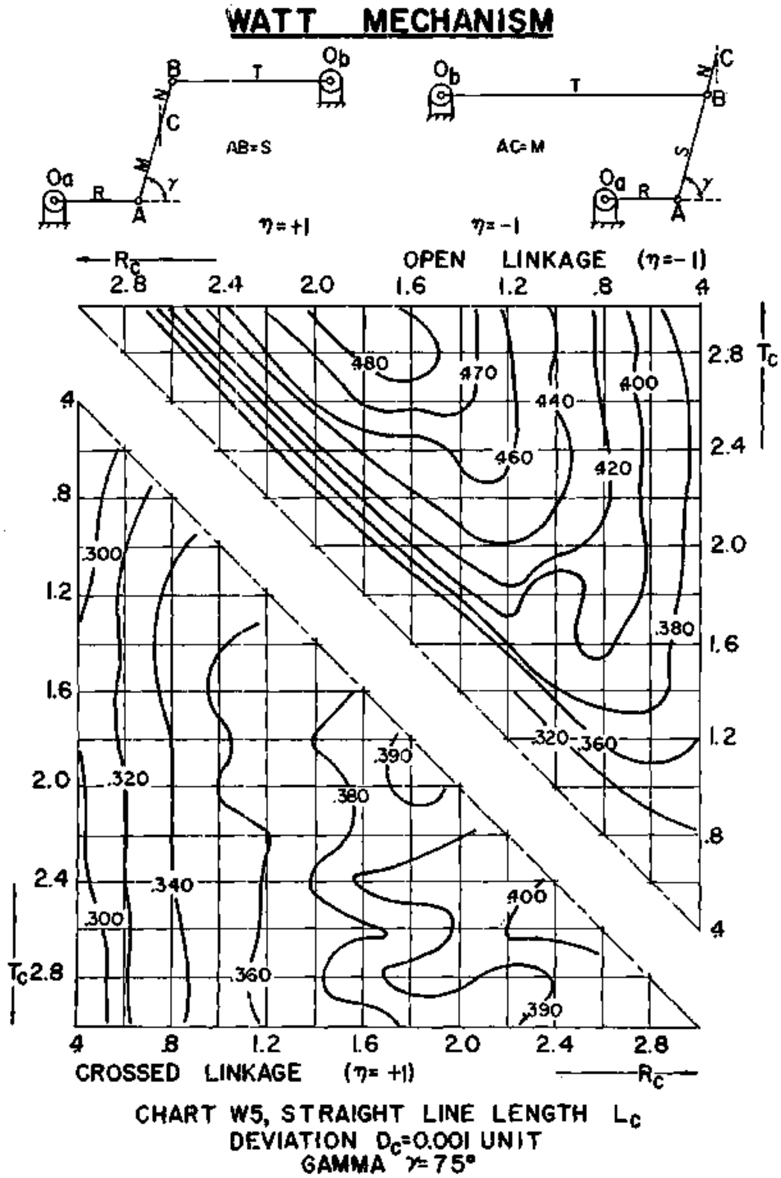


Figure 18. Watt Mechanism Chart W 5.

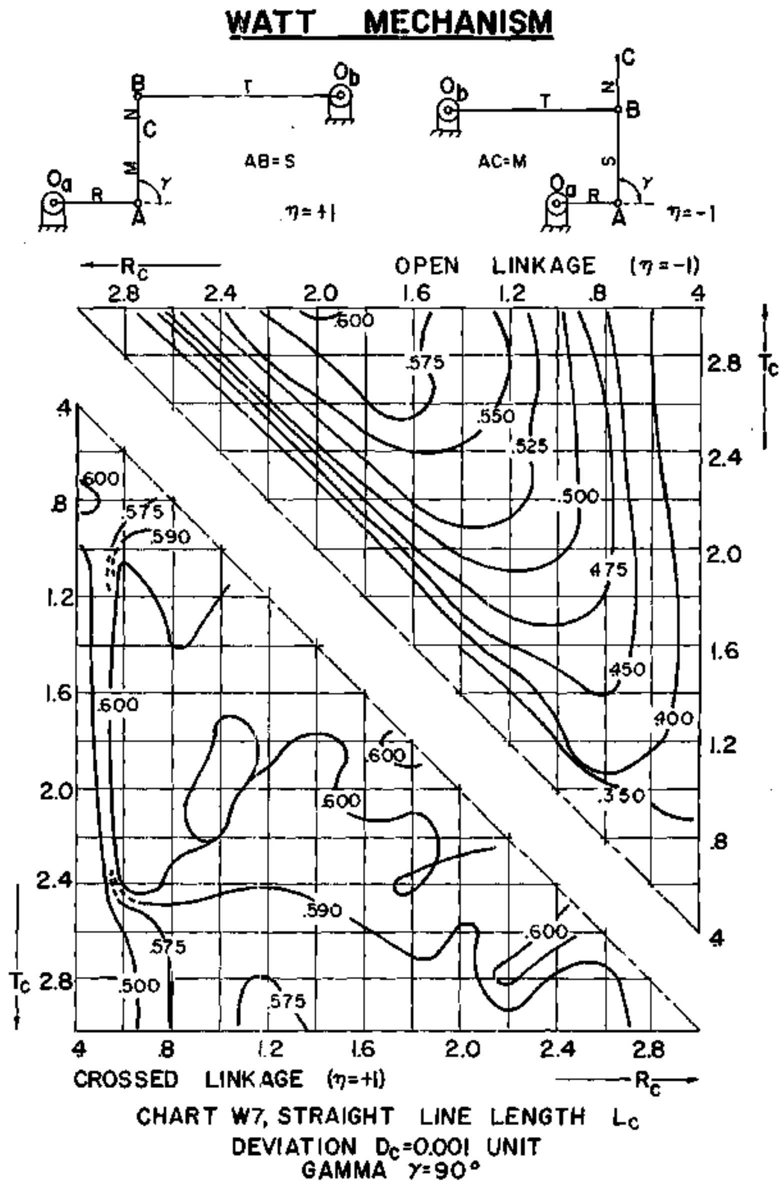


Figure 19. Watt Mechanism Chart W 6.

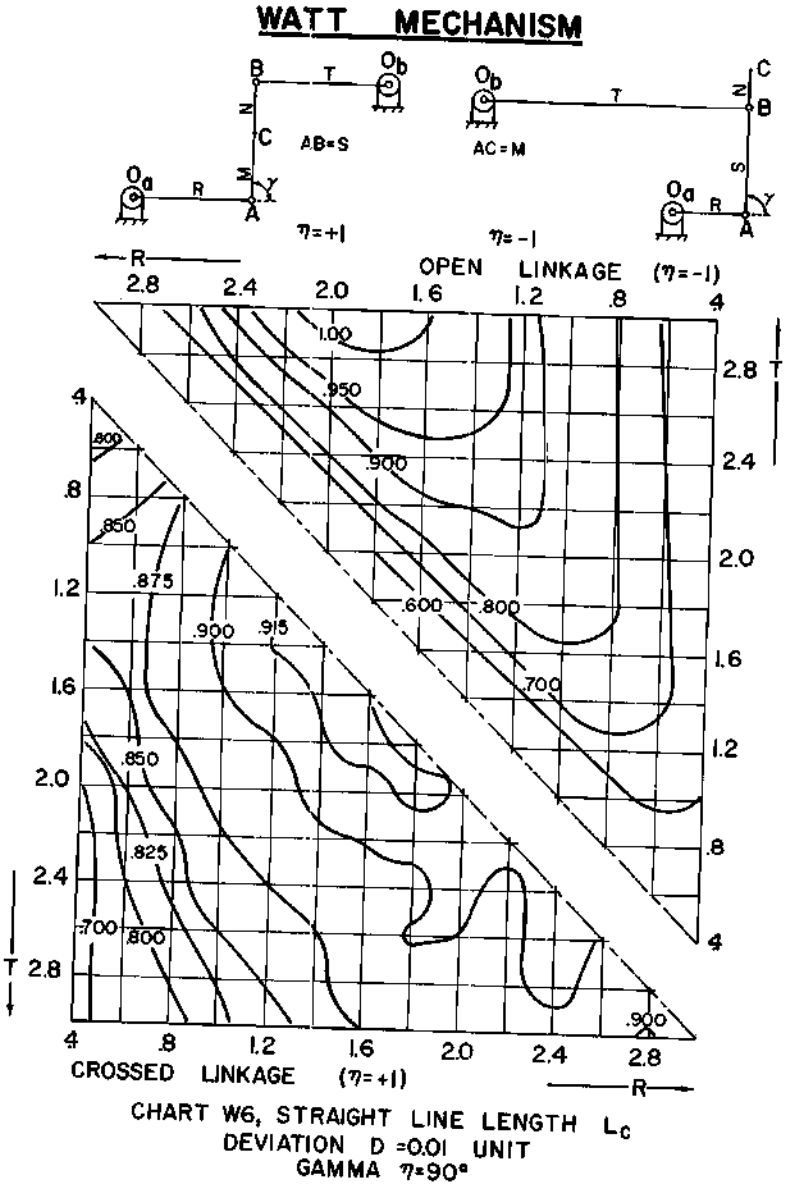


Figure 20. Watt Mechanism Chart W 7.

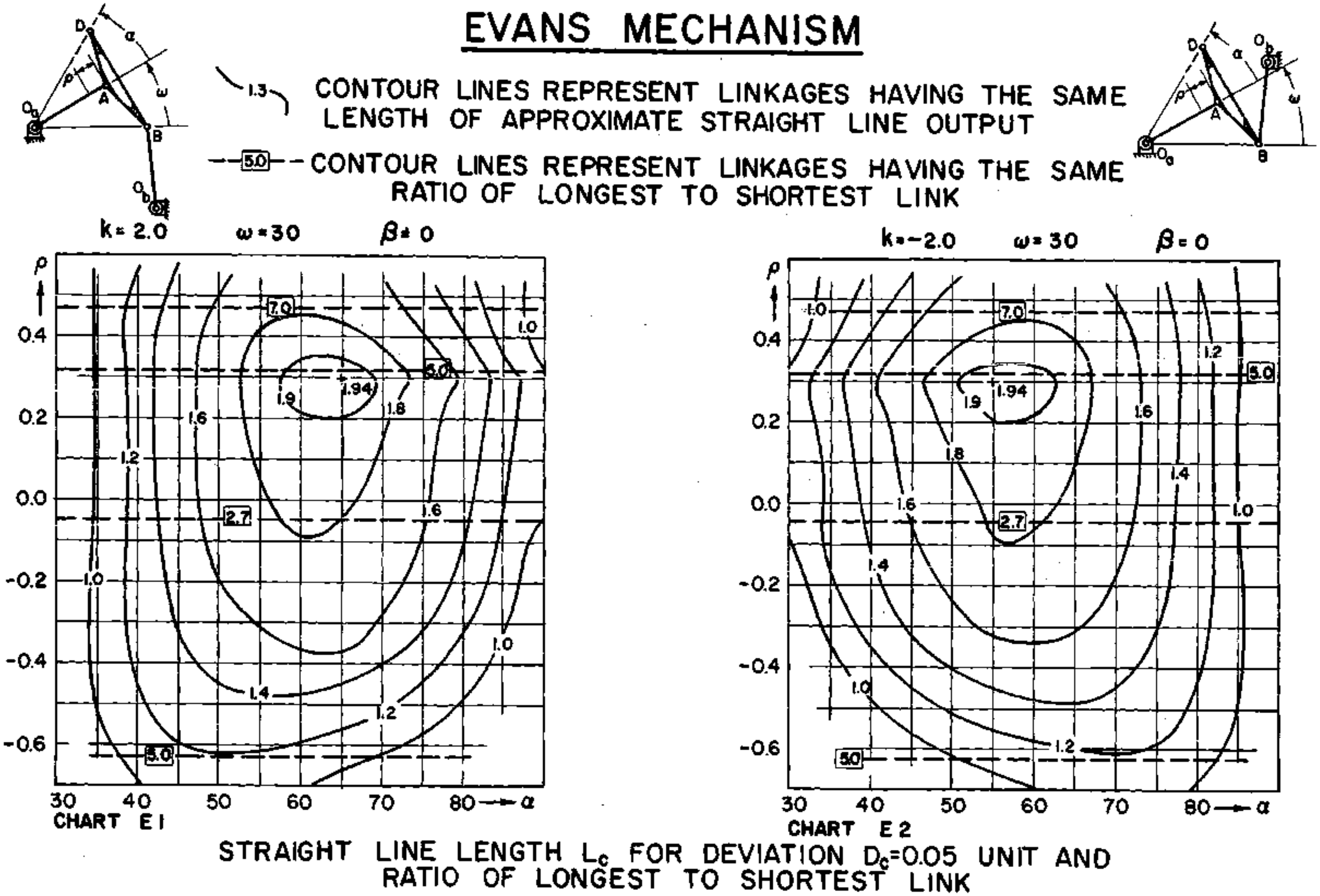


Figure 21. Evans Mechanism Charts E 1 and E 2.

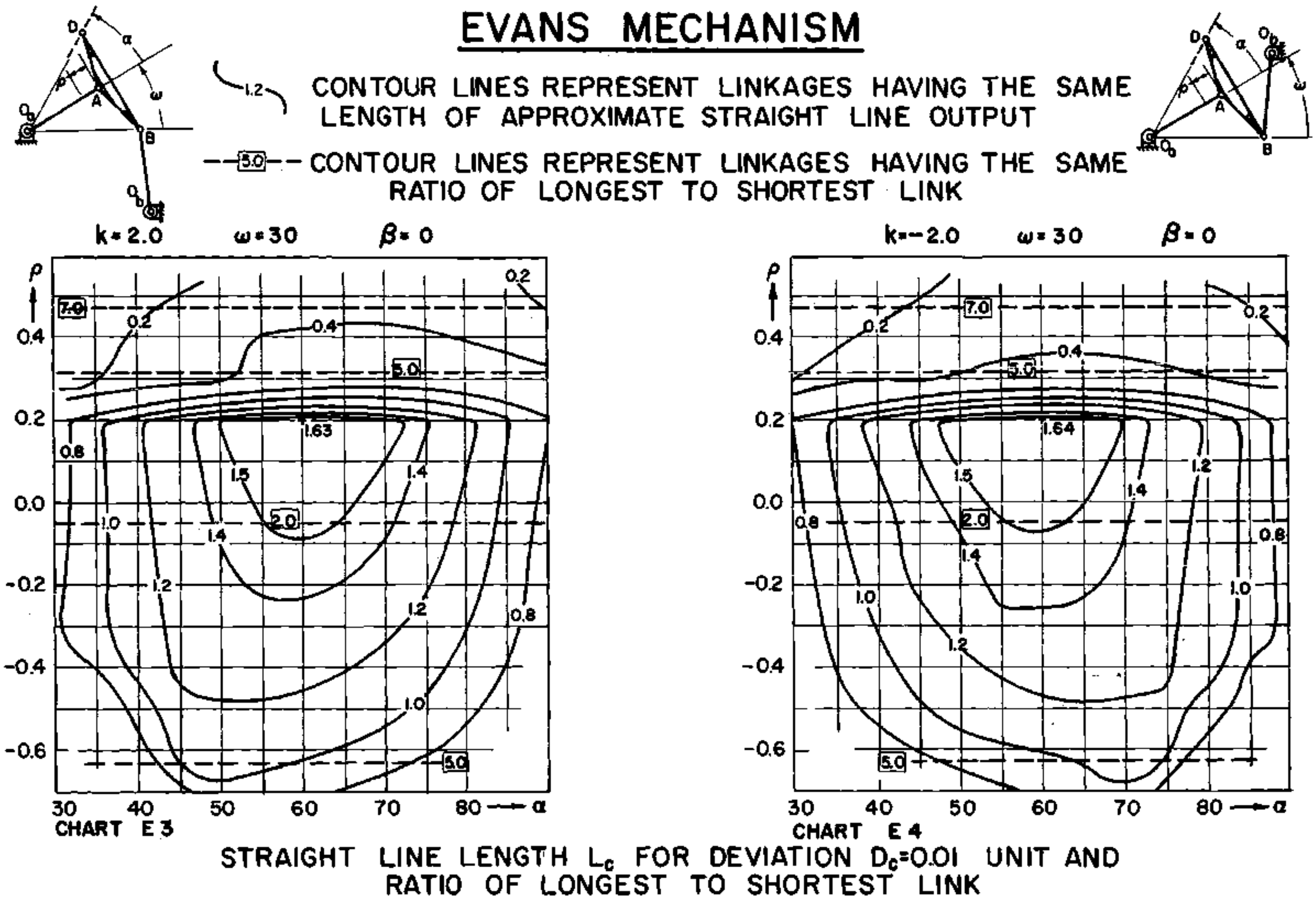
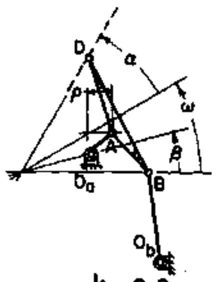


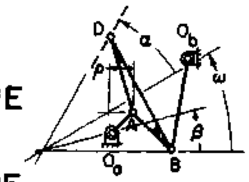
Figure 22. Evans Mechanism Charts E 3 and E 4.

EVANS MECHANISM



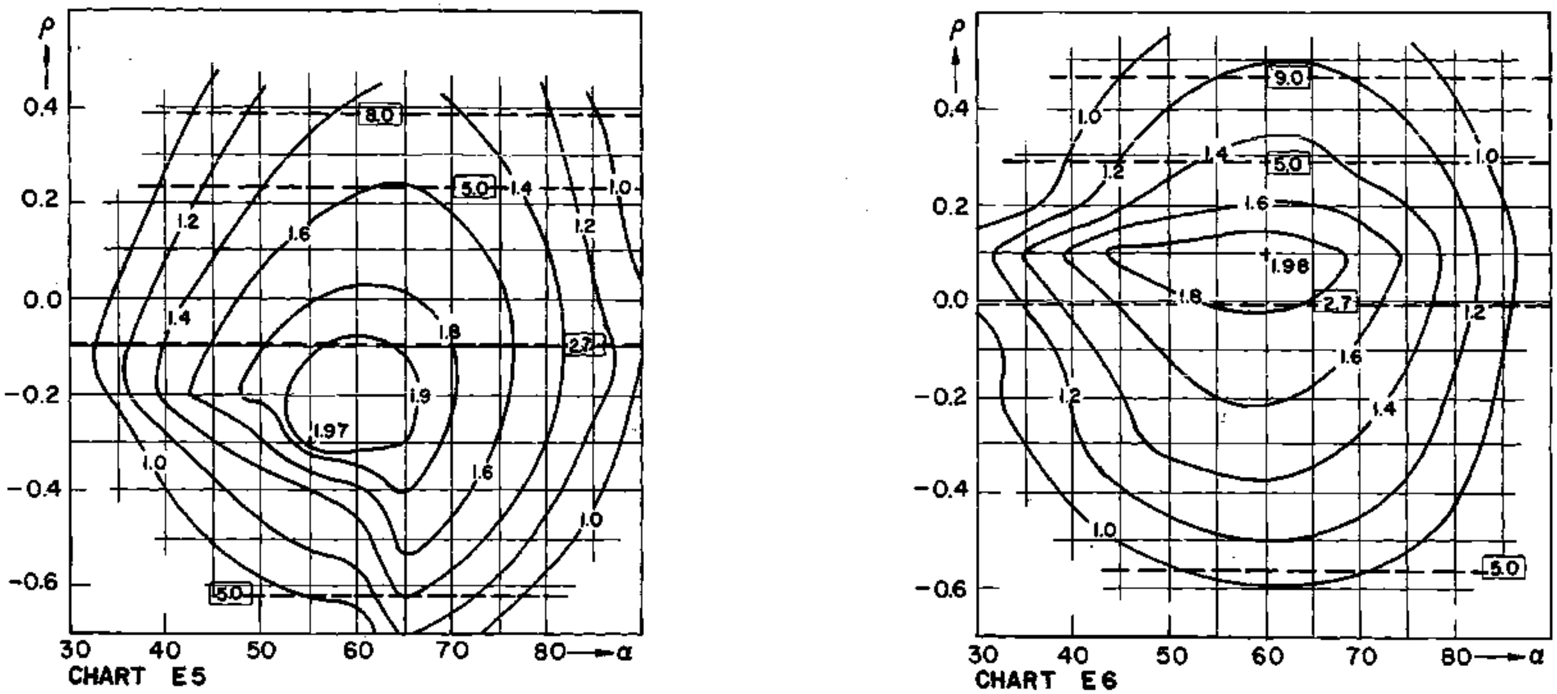
1.3 CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME LENGTH OF APPROXIMATE STRAIGHT LINE OUTPUT

5.0 CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME RATIO OF LONGEST TO SHORTEST LINK



k = 2.0 ω = 30 β = 15

k = −2.0 ω = 30 β = 15



STRAIGHT LINE LENGTH L_c FOR DEVIATION D_c=0.05 UNIT AND RATIO OF LONGEST TO SHORTEST LINK

Figure 23. Evans Mechanism Charts E 5 and E 6.

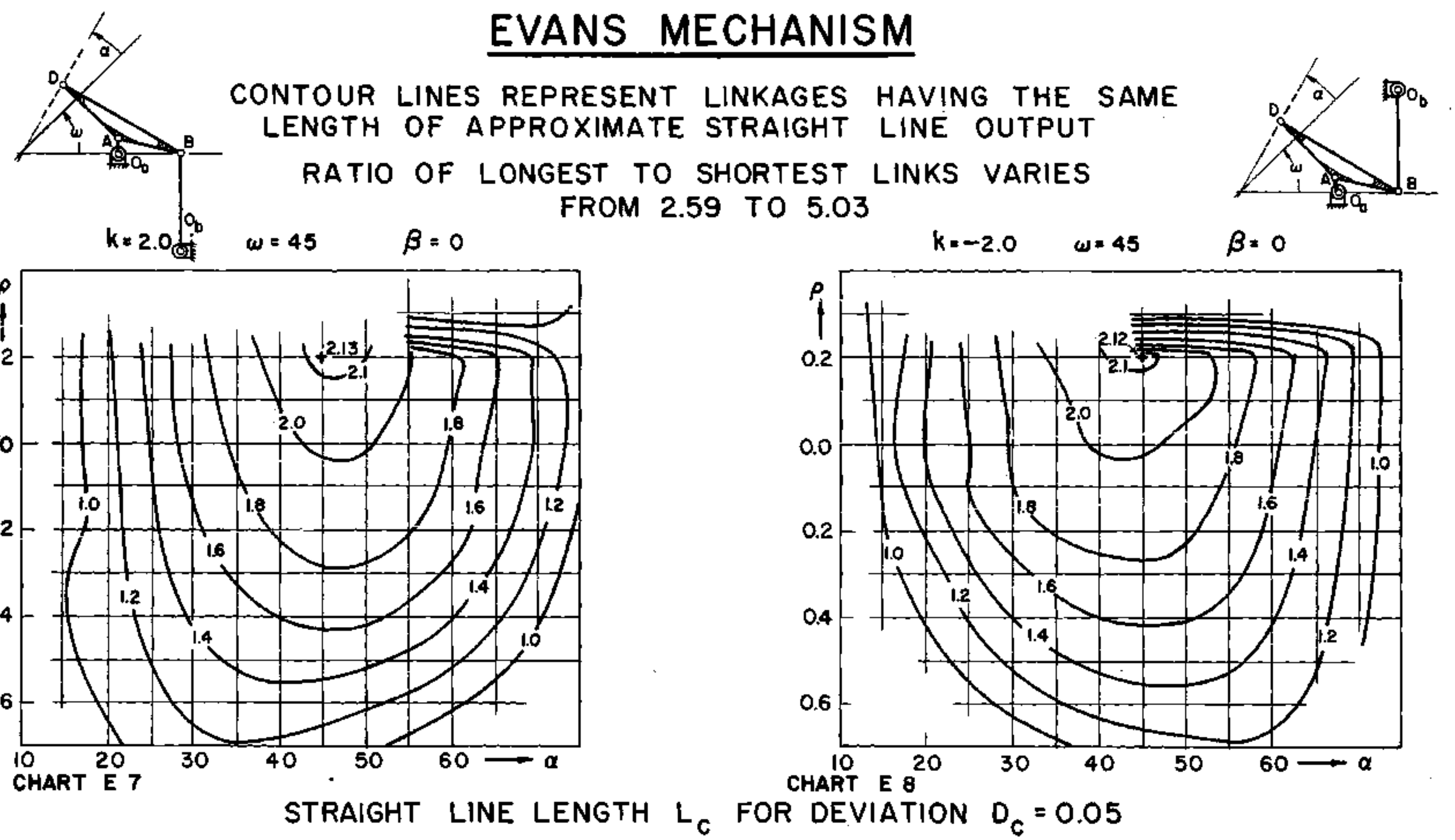
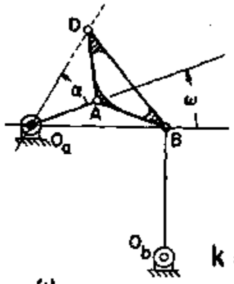


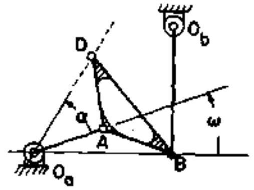
Figure 24. Evans Mechanism Charts E 7 and E 8.

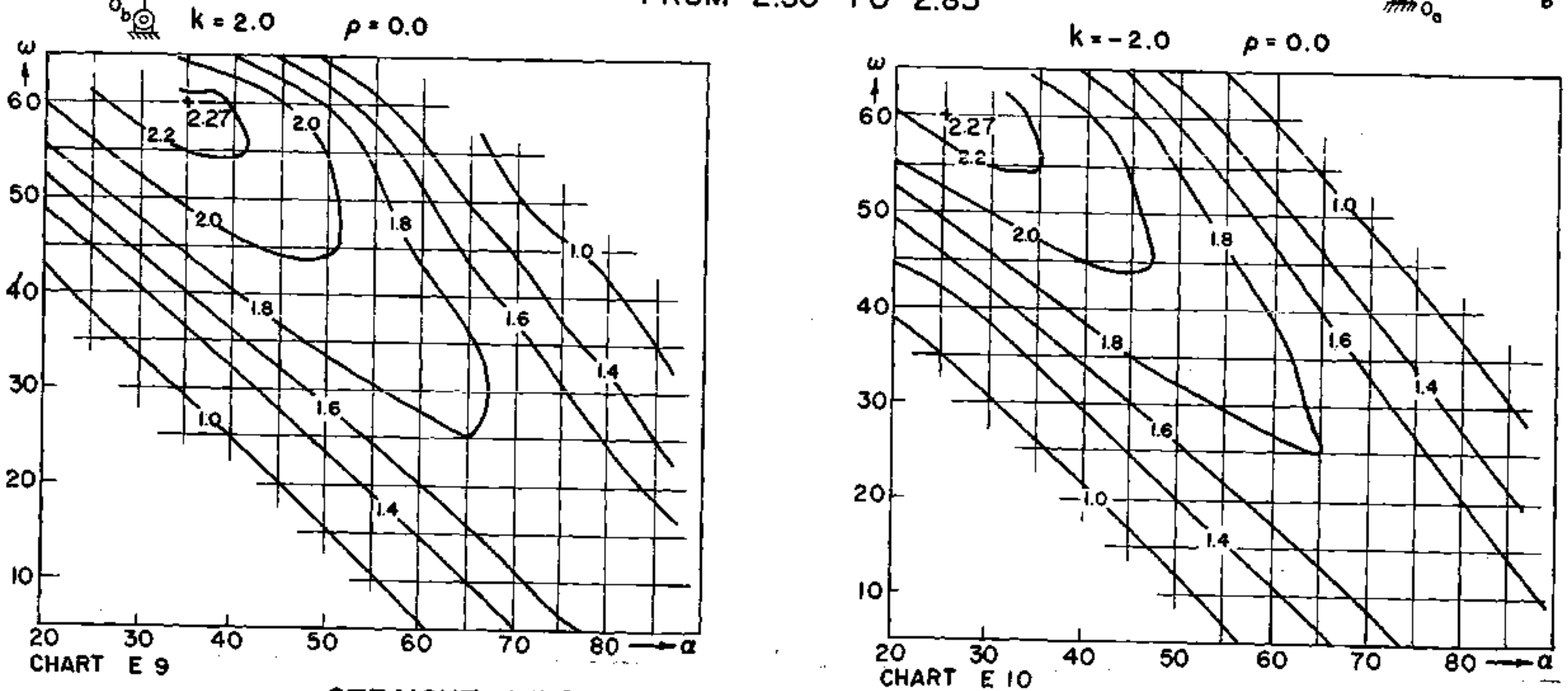
EVANS MECHANISM

CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME LENGTH OF APPROXIMATE STRAIGHT LINE OUTPUT

RATIO OF LONGEST TO SHORTEST LINKS VARIES FROM 2.50 TO 2.83







STRAIGHT LINE LENGTH L_c FOR DEVIATION $D_c = 0.05$

Figure 25. Evans Mechanism Charts E 9 and E 10.

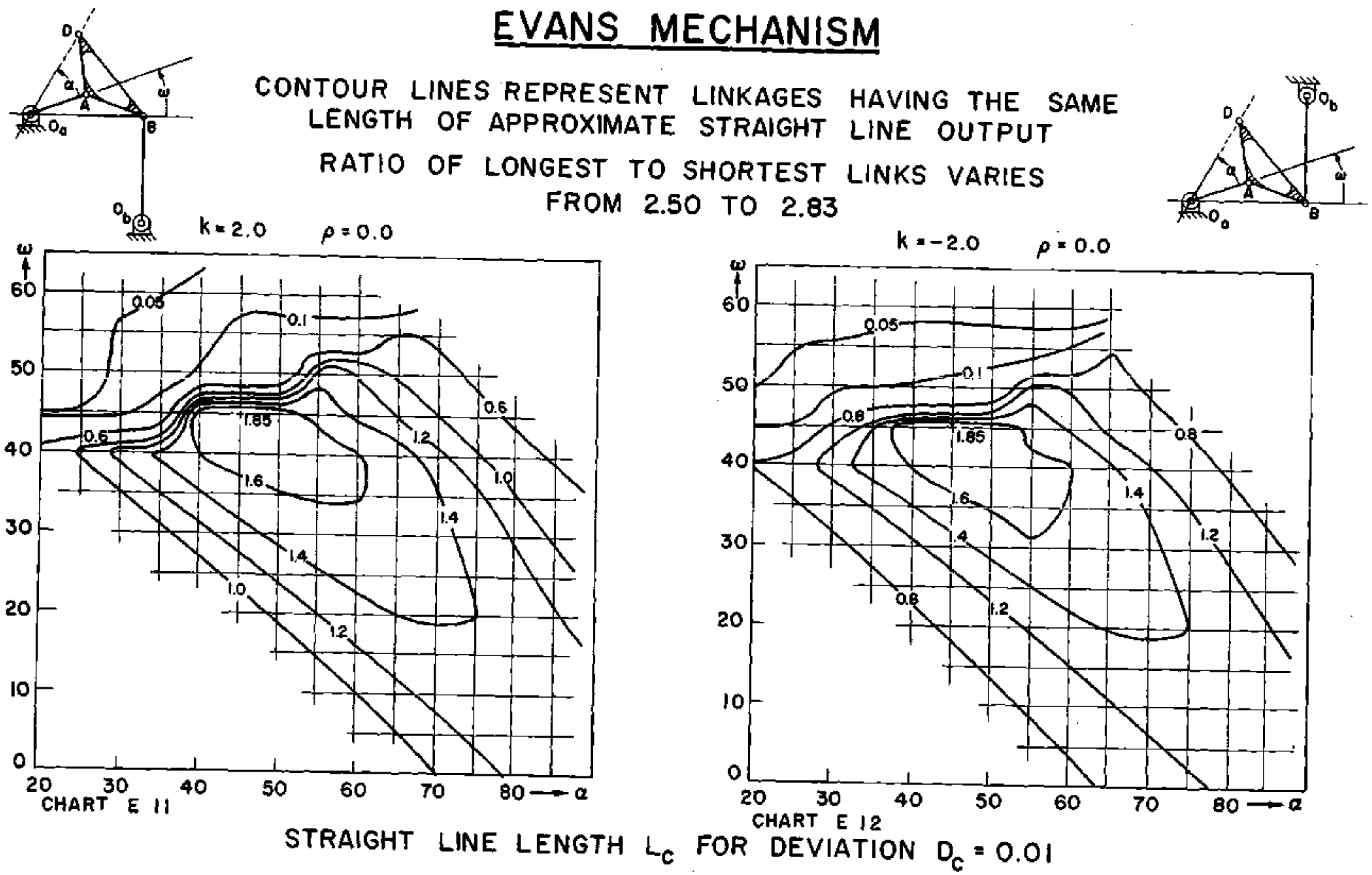
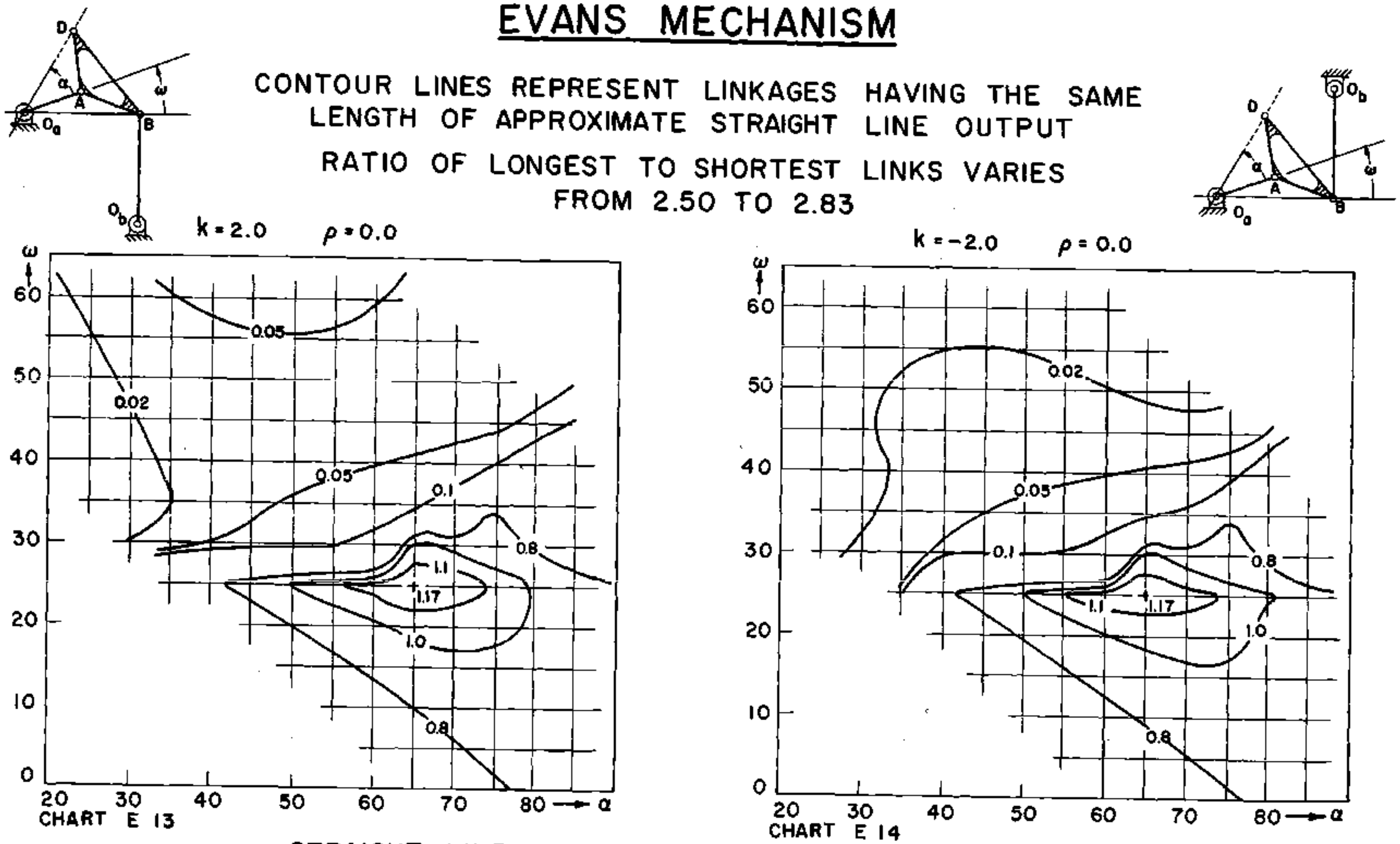


Figure 26. Evans Mechanism Charts E 11 and E 12.

EVANS MECHANISM

CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME LENGTH OF APPROXIMATE STRAIGHT LINE OUTPUT

RATIO OF LONGEST TO SHORTEST LINKS VARIES FROM 2.50 TO 2.83



STRAIGHT LINE LENGTH L_c FOR DEVIATION D_c = 0.001

Figure 27. Evans Mechanism Charts E 13 and E 14.

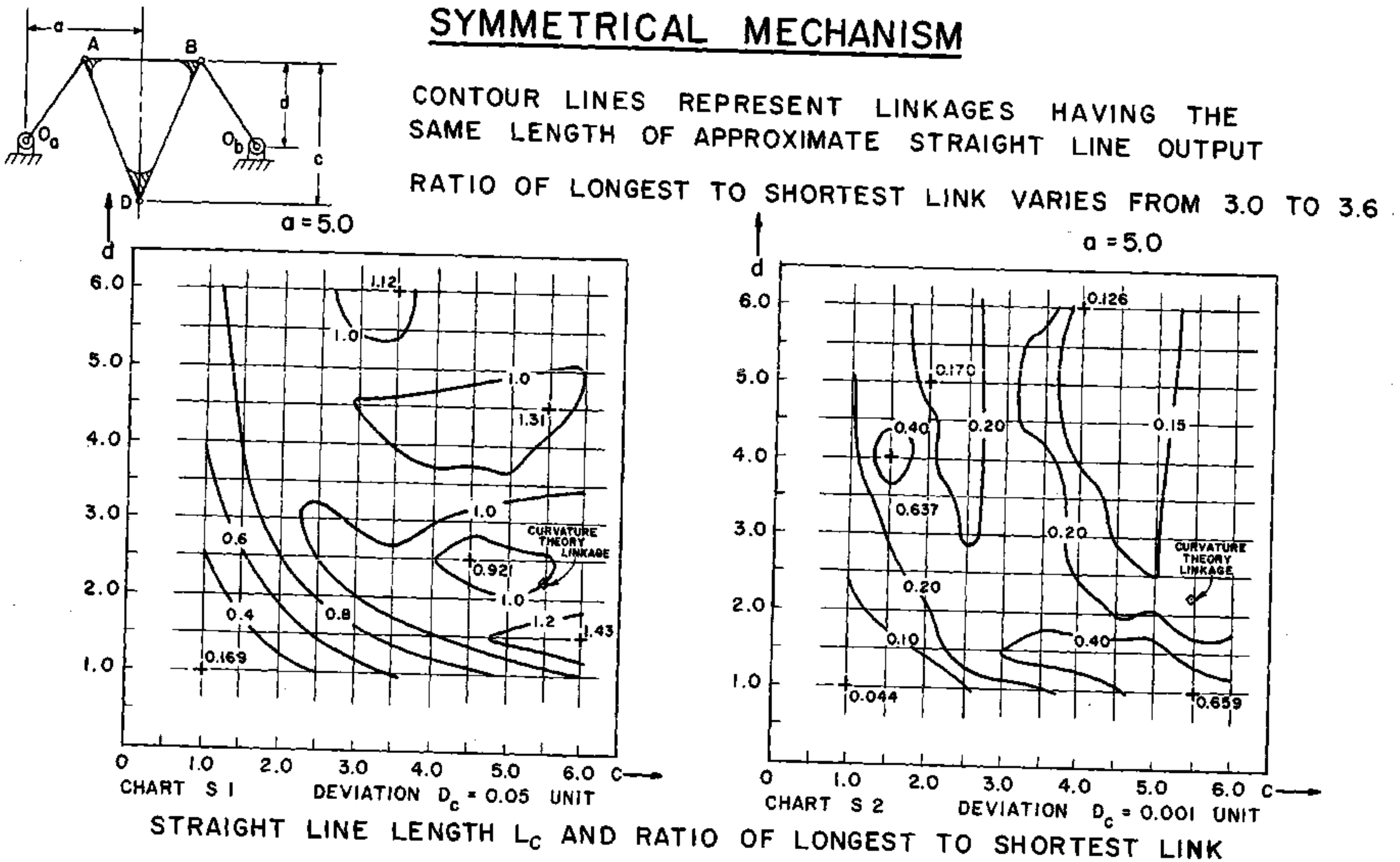


Figure 28. Symmetrical Mechanism Charts S 1 and S 2.

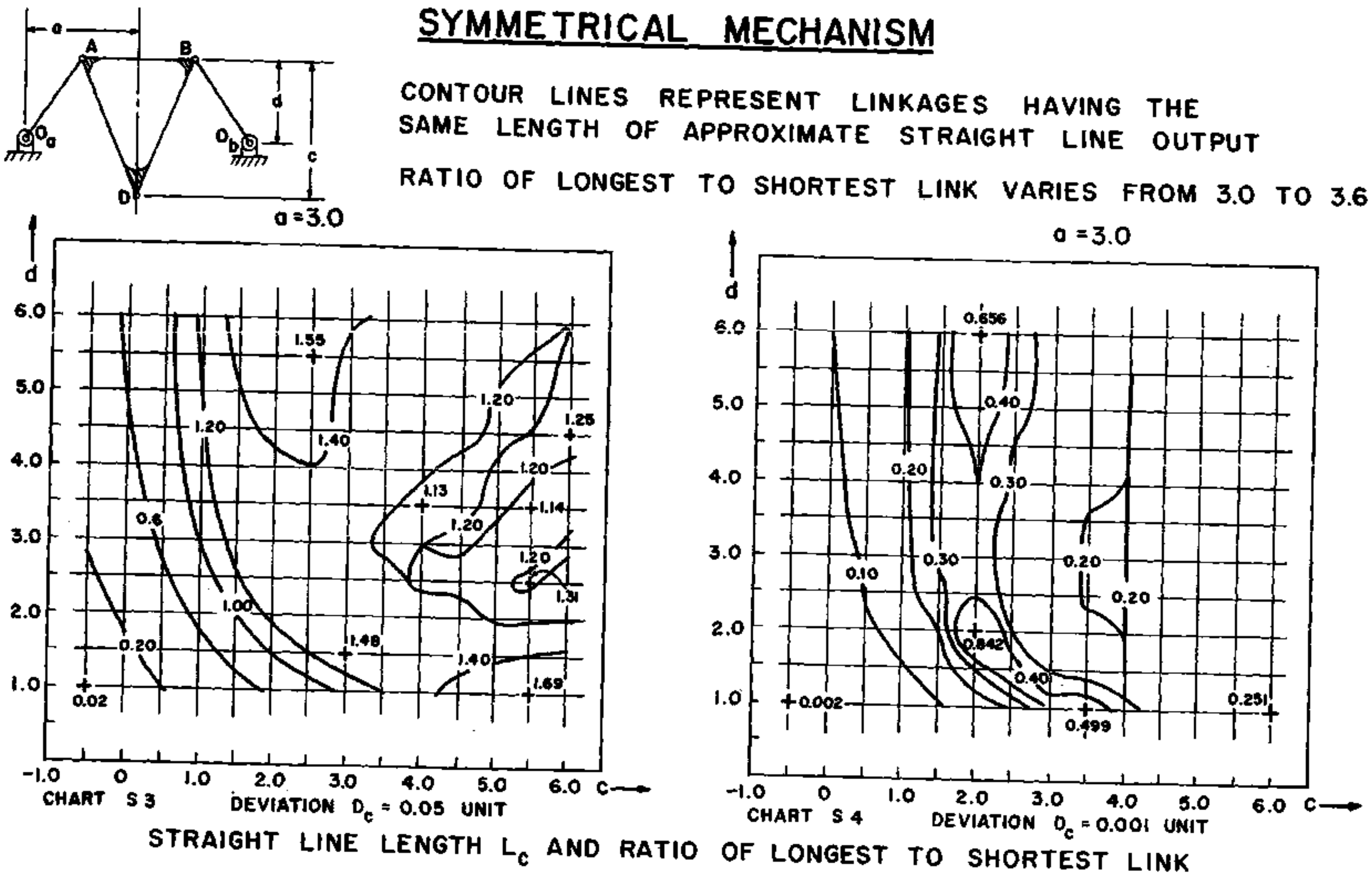


Figure 29. Symmetrical Mechanism Charts S 3 and S 4.

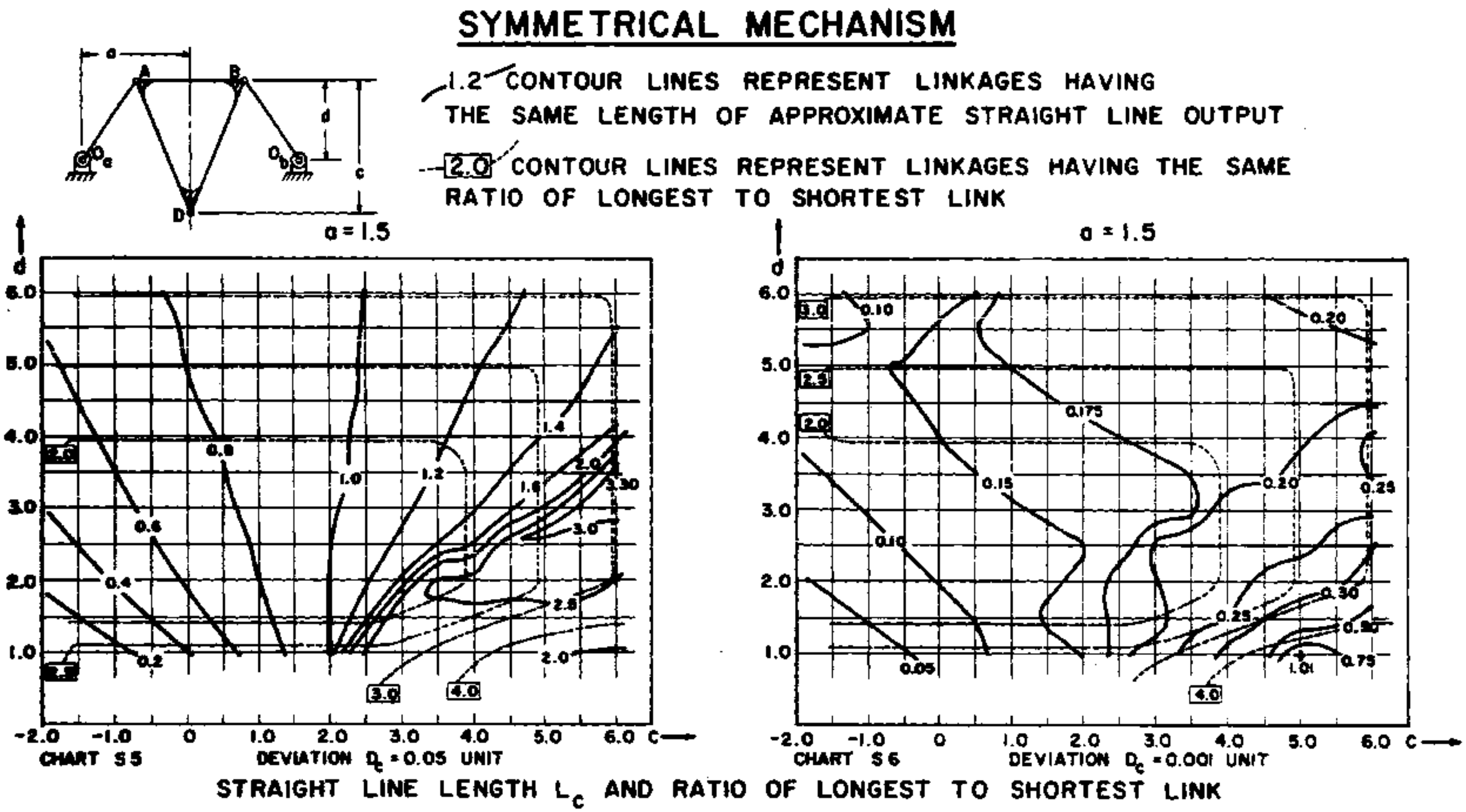


Figure 30. Symmetrical Mechanism Charts S 5 and S 6.

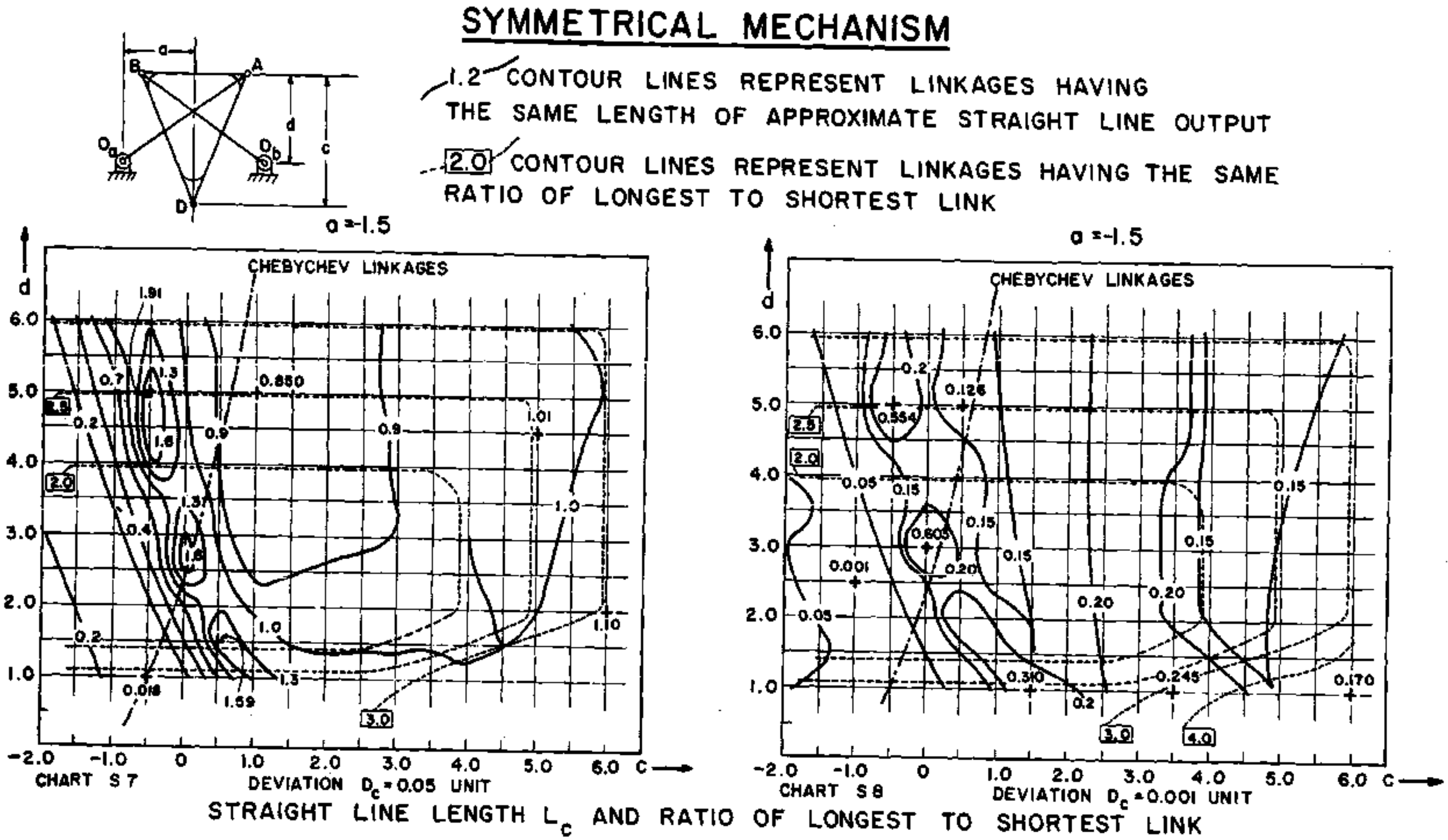
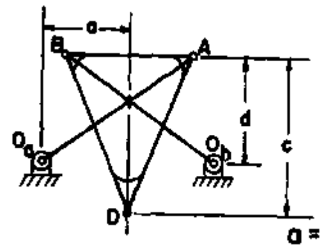


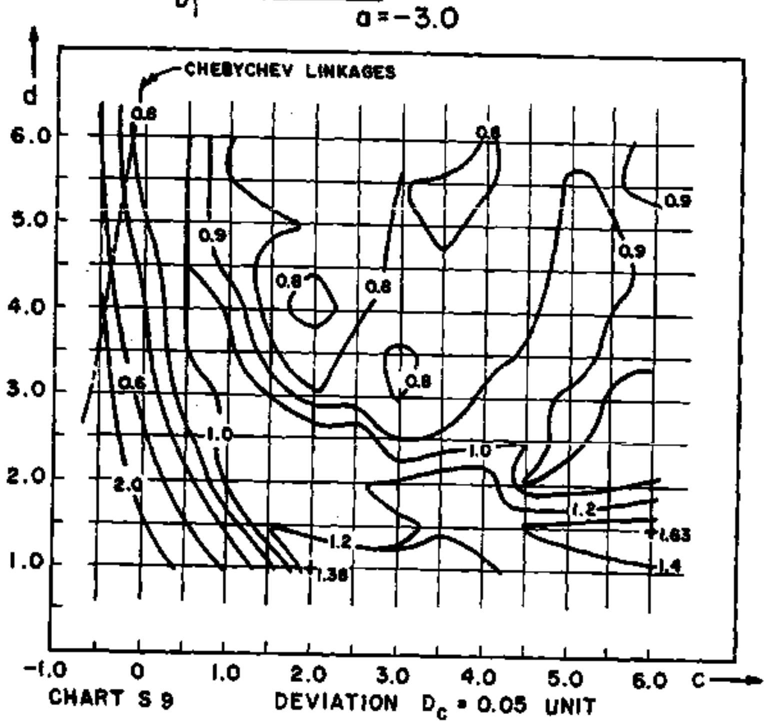
Figure 31. Symmetrical Mechanism Charts S 7 and S 8.

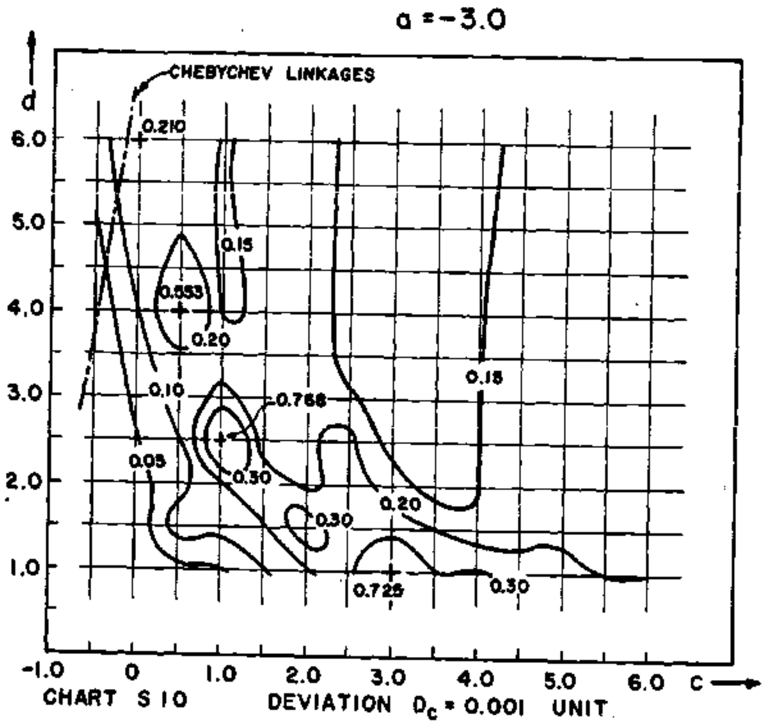


SYMMETRICAL MECHANISM

CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME LENGTH OF APPROXIMATE STRAIGHT LINE OUTPUT

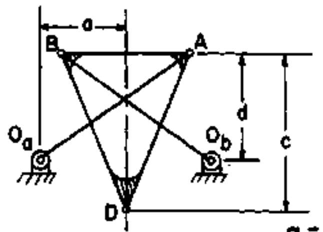
RATIO OF LONGEST TO SHORTEST LINK VARIES FROM 3.0 TO 3.6





STRAIGHT LINE LENGTH L_c AND RATIO OF LONGEST TO SHORTEST LINK

Figure 32. Symmetrical Mechanism Charts S 9 and S 10.



SYMMETRICAL MECHANISM

CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME LENGTH OF APPROXIMATE STRAIGHT LINE OUTPUT

RATIO OF LONGEST TO SHORTEST LINK VARIES FROM 3.0 TO 3.6

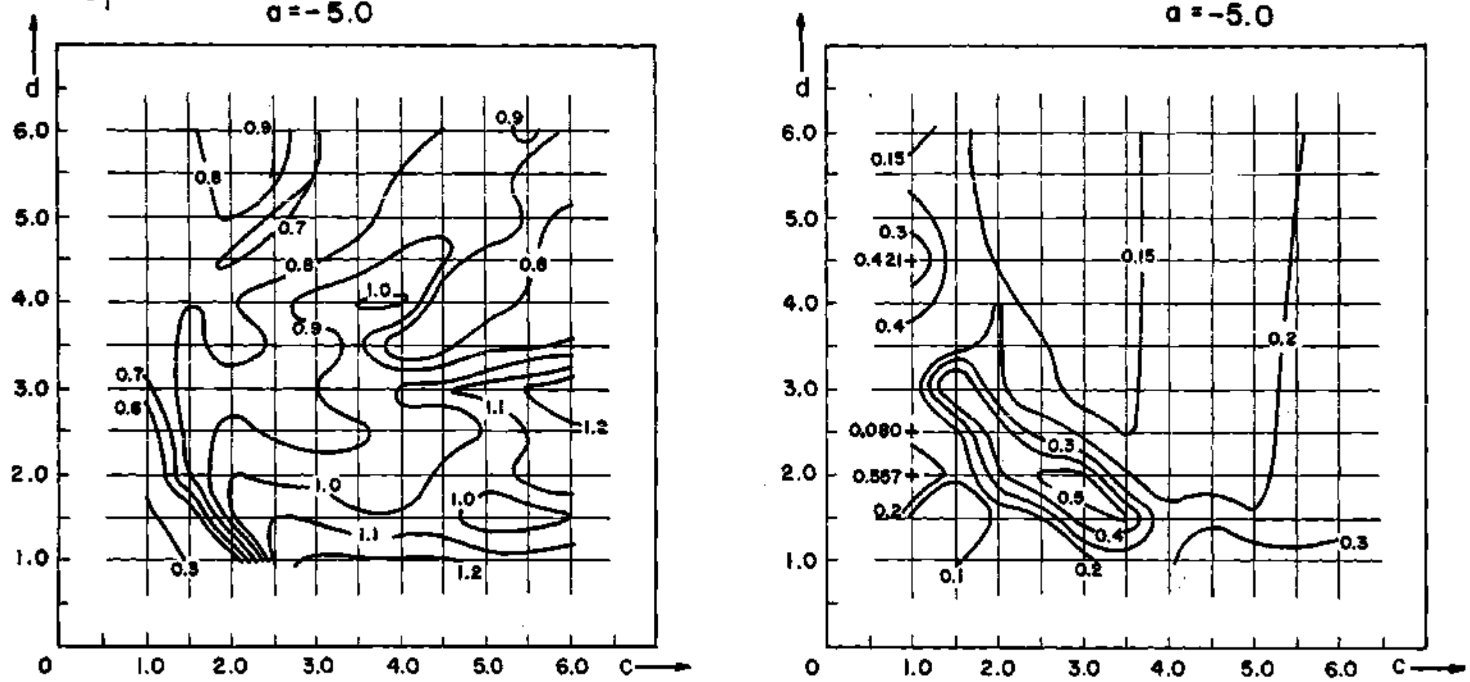


CHART S 11 DEVIATION D_c = 0.05

CHART S 12 DEVIATION D_c = 0.001

STRAIGHT LINE LENGTH L_c AND RATIO OF LONGEST TO SHORTEST LINK

Figure 33. Symmetrical Mechanism Charts S 11 and S 12.

APPENDIX C

SAMPLE COMPUTER PROGRAM

Included in Appendix C is a sample computer program for the Evans mechanism with a representative sample output. The computer programs for the Watt and Symmetrical mechanisms are very similar and it was therefore considered not necessary to include them here.

```
COMMENT STUDY OF THE DEVIATION FROM EXACT STRAIGHT LINE  MOTION AS
        PRODUCED BY EVANS TYPE LINKAGES.  THE OUTPUT WILL GIVE
        THE PRESSURE ANGLE, TYPE OF MECHANISM, THE LENGTH OF THE
        STRAIGHT LINE OUTPUT FOR THREE DIFFERENT ACCURACY
        SPECIFICATIONS --- JAMES HIEGEL , M.E., G.I.T., JULY  1964      $
INTEGER I,J,L,Z,F,FAT(),AM,AG,MEN,MAD,AT,MA                             $
ARRAY             U(3),V(3),VP(2),UP(2),UU(2),VV(2),VVP(2),UUP(2),
        UPU(2),VPV(2),DR(6),UM(6),GA(6),GB(6),IP(6),IS(6),CH(6),
        DL(330),NS(330),FAT(4)                                          $
        G=1.7453293**-2     $      Z=0   $           W=(180.0.G)        $ 12
PROCEDURE ANGLE(X,Y$V)                                                  $ 13
BEGIN   S=ABS(X/0.017453293)$          T=ABS(Y/0.017453293)             $ 14
EITHER  IF (X.Y) GTR 0.0                                                $ 15
BEGIN   V=ABS(S-T)              $      RETURN                END        $ 16
        OTHERWISE                                                       $ 17
EITHER  IF (S+T) GTR 90.0                                               $ 18
BEGIN   V=ABS(180.0-(S+T))      $      RETURN                END        $ 19
        OTHERWISE                                                       $ 20
BEGIN   V=S+T                   $      RETURN                END        $ 21
        END ANGLE ()                                                    $ 22
SUBROUTINE PATH                                                         $
BEGIN   EITHER IF MA LSS 0               $          AT=3 $ OTHERWISE    $
         AT=0                                                           $
        IF ((-MA)(F-180)) EQL 1.0                                       $
BEGIN   FOR AM=(AT+1,1,AT+3)                                            $
BEGIN   DR(AM)=UM(AM)=GA(AM)=GB(AM)=IP(AM)=IS(AM)=CH(AM)=0.0    END     $
        GBT=GAT=90.0            $     DOT= 0.0                          $
      Z=MEN=SV=SAVE=SU=GIN=SIR=SOT=D=0.0 $  FAT(AT+1)=0  $L=1  END      $
        FOR Z=(Z,AG,180)                                                $
BEGIN   PHIR=PPHIR+(G.Z)                                                $
        IF PHIR LSS 0.0                  $          PHIR=PHIR+(2.0.W)   $
        PHI=PHIR/G                                                      $
        IF (MAD EQL 2) AND (PPHIR GTR W)                                $
BEGIN   IF PHIR LSS W                    $          GO TO ADD END       $
        IF (MAD EQL 2) AND (PPHIR LSS W)                                $
BEGIN   IF PHIR GTR W                    $          GO TO ADD END       $
        IF MAD EQL 1                                                    $
ADD..   BEGIN EITHER IF AG GTR 0                                        $
BEGIN   IF PHIR GTR SPHIR     $    (Z=Z-AG     $    RETURN END END      $
     OTHERWISE                                                          $
BEGIN   IF PHIR LSS SPHIR     $    (Z=Z-AG     $    RETURN END END END  $
        LA=SIN(PHIR)                     $          F=F+MA              $
        LB = COS(PHIR)-(Q/R)                                            $ 117
        LC = ((Q.Q+R.R+T.T-S.S)/(2.0.R.T)) -(Q/T)COS(PHIR)              $ 118
        LD = SQRT(LA.LA+LB.LB-LC.LC)                                    $ 126
        SIR = (2.0)ARCTAN((LA-CA.LD)/(LB+LC))                           $ 127
EITHER  IF (SIR GTR W/2.0) AND (SIR LEQ W)       $          SIRI=SIR-W  $ 130
OR      IF (SIR LSS -W/2.0) AND (SIR GEQ -W)     $          SIRI=SIR+W  $ 131
        OTHERWISE                                $          SIRI=SIR    $ 132
        AX = R.COS(PHIR)        $          AY = R.SIN(PHIR)             $ 133
        BX = T.COS(SIR) + Q $              BY = T.SIN(SIR)              $ 134
        IF SIR LSS 0.0          $    SIR=SIR+2.0.W  $    SI=(SIR-SSIR)/G  $
        IF ABS(SI) GTR 200.0$ SI=(360.0-(SIGN(SI))(SI))(SIGN(W-SIR))    $
BEGIN   IK=ARCTAN((BY-AY)/(BX-AX))                                      $ 135
EITHER  IF (BX-AX) LSS 0.0  $              KIR=IK+W                     $ 136
        OTHERWISE           $              KIR=IK                       $ 137
```

```
        IF KIR LSS 0.0       $          KIR=KIR+2.0.W        END          $ 138
        KI=KIR/G                                                          $
WIFE..  X=((R)(COS(PHIR)))+((M)(COS(ER+KIR)))                             $
        Y=((R)(SIN(PHIR)))+((M)(SIN(ER+KIR)))                             $ 140
       US=(((X)(COS(THR)))-((Y)(SIN(THR)))+(U(I)))                        $ 141
        VS=((X)(SIN(THR)))+((Y)(COS(THR)))+(V(I))                         $ 142
        SU=((US-U(3))(COS(ALR))-(VS-V(3))(SIN(ALR)))/UN                   $
        SV=-(US-U(3))(SIN(ALR))+(VS-V(3))(COS(ALR))                       $
        D=ABS(SV/UN)                                                      $ 145
        IF ABS(Z) EQL 1.0    $          GIN=SIGN(SU-SOT)                  $
        IF PU NEQ 1.0                                                     $
BEGIN   IF F EQL 180    $    BEGIN SOT=SU    $     IF D GTR 0.001         $
BEGIN   FR=-FR     $     E=-E       $ PU=1.0 $ GO TO WIFE     END END END $
        IF ((GIN)(SU-SAVE) LSS 0.0)                                       $
EITHER  IF MEN EQL  0        $  (SU=SAVE     $  DOT=1.0  $  GO TO DUT)    $
OR      IF MEN EQL 1         $  (SU=SAVE  $  DOT= 1.0  $  GO TO GUT)      $
OTHERWISE                    $  (SU=SAVE  $  DOT =1.0  $  GO TO MUT)      $
        DL(E)=SV/UN                                                       $ 155
        NS(E)=SU-SOT         $          SAVE=SU                           $ 156
        ANGLE((PHHI+Z.G),IK$GAB)                                          $ 157
        ANGLE(SIRI,IK$GBA)                                                $ 158
        IF GAB LSS GAT                  $          GAT=GAB                $
        IF GBA LSS GBT                  $          GBT=GBA                $
        SWITCH MEN,(BAD,GOOD)                                             $ 159
        IF D GTR 0.001                                                    $ 160
DUT..   BEGIN UM(AT+1)=SU-SOT     $     MEN=1       $     CH(AT+1)=KI-KKI $
        GA(AT+1)=GAT         $     GB(AT+1)=GBT$  DR(AT+1)=D              $
        IP(AT+1)=Z                $     IS(AT+1)=SI            END        $
BAD..   IF D GTR 0.01                                                     $
GUT..   BEGIN UM(AT+2)=SU-SOT     $     MEN=2       $     CH(AT+2)=KI-KKI $
        GA(AT+2)=GAT              $     GB(AT+2)=GBT$  DR(AT+2)=D         $
        IP(AT+2)=Z                $     IS(AT+2)=SI            END        $
GOOD..  IF D GTR 0.05                                                     $
MUT..   BEGIN UM(AT+3)=SU-SOT     $     FAT(AT+1)=F $  CH(AT+3)=KI-KKI    $
        GA(AT+3)=GAT              $     GB(AT+3)=GBT$  DR(AT+3)=D         $
        IP(AT+3)=Z                $     IS(AT+3)=SI                       $
        RETURN                                                 END        $
                                                    END        END        $
        FOR K = -2.0, 2.0                                                 $
        FOR OMGA = 30.0                                                   $
        FOR BETA = 15.0                                                   $
        FOR RHO = -0.4                                                    $
        FOR ALFA =  75.0                                                  $
BEGIN   BER=(G)(BETA)  $ ALR=(G)(ALFA+OMGA)  $       OMR=(G)(OMGA)        $
        IF    OMGA EQL BETA              $  GO TO JOB                     $
        IF ABS (OMGA-BETA) GTR  45.0               $  GO TO JOB           $
        I=1          $          J=2                                       $ 23
        LM=(SIN(OMR)-(RHO)SIN(2.0(BER)-(OMR)))/
        (COS(OMR)-(RHO)COS(2.0(BER)-(OMR)))                               $
EITHER  IF BER EQL 0.0        $      V(1) = 0.0                           $
        OTHERWISE                                                         $
        V(1)=(((2.0)SIN(OMR))-(2.0.LM)COS(OMR))/(1.0-(LM)/(TAN(BER))) $
        U(1)=(2.0)(SIN(OMR)-(LM)COS(OMR))/(TAN(BER)-LM)                   $
        VP(1)=SIN(OMR)+(RHO)SIN((2.0)(BER)-(OMR))                         $
        UP(1)=COS(OMR)+(RHO)COS((2.0)(BER)-(OMR))                         $
        V(2)=-K                                                           $
```

```
        U(2)=1+COS(OMR)                                                   $
        VP(2)=0                                                           $
        UP(2)= 2.0.COS(OMR)                                               $
        U(3)=(2.0)COS(ALR)COS((ALR)-(OMR))                                $
        V(3)=(2.0)SIN(ALR)COS((ALR)-(OMR))                                $
        UU(I)=U(J)-U(I)                                                   $ 51
        VV(I)=V(J)-V(I)                                                   $ 52
        UUP(I)=UP(J)-UP(I)                                                $ 53
        VVP(I)=VP(J)-VP(I)                                                $ 54
        UPU(I)=UP(I)-U(I)     $          UPU(J)=UP(J)-U(J)                $ 55
        VPV(I)=VP(I)-V(I)     $          VPV(J)=VP(J)-V(J)                $ 56
        Q =   SQRT((UU(I))(UU(I))+(VV(I))(VV(I)))                         $ 57
        R=SQRT((UPU(I))(UPU(I))+(VPV(I))(VPV(I)))                         $
        S=SQRT((UUP(I))(UUP(I))+(VVP(I))(VVP(I)))                         $
        T=SQRT((UPU(J))(UPU(J))+(VPV(J))(VPV(J)))                         $
EITHER  IF UU(I) LSS 0.0      $         THR=ARCTAN(VV(I)/UU(I))+W         $ 60
        OTHERWISE             $         THR=ARCTAN(VV(I)/UU(I))           $ 61
        IF THR LSS 0.0        $          THR=THR+2.0.W                    $ 62
        XXP = UPU(I)COS(THR) + VPV(I)SIN(THR)                             $ 63
        YYP =-UPU(I)SIN(THR) + VPV(I)COS(THR)                             $ 64
        XPX = UPU(J)COS(THR) + VPV(J)SIN(THR)                             $ 65
        YPY =-UPU(J)SIN(THR) + VPV(J)COS(THR)                             $ 66
        PHHI=ARCTAN(YYP/XXP)                                              $ 67
EITHER  IF XXP LSS 0.0        $          PPHIR=PHHI+W                     $ 68
        OTHERWISE             $          PPHIR=PHHI                       $ 69
        IF PPHIR LSS 0.0      $          PPHIR=PPHIR+2.0.W                $ 70
        PPHI=PPHIR/G                                                      $ 71
        PPSI=ARCTAN(YPY/XPX)                                              $ 72
EITHER  IF (XPX) LSS 0.0      $          SSIR=PPSI+W                      $ 73
        OTHERWISE             $          SSIR=PPSI                        $ 74
        IF SSIR LSS 0.0       $          SSIR=SSIR+2.0.W                  $ 75
        SSI=SSIR/G                                                        $ 76
BEGIN   IKKIR=ARCTAN((YPY-YYP)/(XPX +Q-XXP))                              $ 77
EITHER  IF (XPX-XXP+Q) LSS 0.0$          KKIR=IKKIR+W                     $ 78
        OTHERWISE             $          KKIR=IKKIR                       $ 79
        IF KKIR LSS 0.0       $          KKIR=KKIR+2.0.W       END        $ 80
        ANGLE(PHHI,IKKIR$GGA)            $         KKI=KKIR/G             $
        ANGLE(PPSI,IKKIR$GGB)            $         GAT=GGA $  GBT=GGB     $
        XJ=(U(3)-U(I))COS(THR)+(V(3)-V(I))SIN(THR)                        $
        YJ=-(U(3)-U(I))SIN(THR)+(V(3)-V(I))COS(THR)                       $
        MX = XJ-R.COS(PPHIR)                                              $
        MY = YJ-R.SIN(PPHIR)                                              $
        M = SQRT(MX.MX + MY.MY)                                           $
EITHER  IF MX LSS 0.0         $          ER=ARCTAN(MY/MX)+W               $
        OTHERWISE             $          ER=ARCTAN(MY/MX)                 $
        IF ER LSS 0.0         $          ER=ER+2.0.W                      $
        ER=ER-KKIR            $          E=ER/G                           $
        N=SQRT(ABS(S.S+M.M-2.0.M.S.COS(ER)))                              $
        MM = MAX(Q,R,S,T,((M+N)/2.0))                                     $ 90
        NN = MIN(Q,R,S,T,(M+N))                                           $ 91
        RAT=MM/NN $     IF RAT GTR 10.0  $         GO TO JOB              $
        UU1=(VPV(I)+(U(I))TAN(THR)-(UP(I))(VVP(I)/UUP(I)))/
        (TAN(THR)-(VVP(I)/UUP(I)))                                        $
        VV1=(UU1)(VVP(I)/UUP(I))+VP(I)-(VVP(I)/UUP(I))(UP(I))             $
EITHER IF (SQRT((U(I)-UU1)(U(I)-UU1)+(V(I)-VV1)(V(I)-VV1))+
           SQRT((U(J)-UU1)(U(J)-UU1)+(V(J)-VV1)(V(J)-VV1))) GTR
```

```
         (Q+0.000001)                                                            $
         CCA=1.0                 $  OTHERWISE           $  CCA=-1.0              $
         UN=       (Q+R+S+T+((M+N)/2.0))/5.0                                     $
EITHER   IF (Q+S) GTR (T+R)                                                      $
BEGIN    SA = MAX(Q,S)  $  SB=Q+S-SA  $  SC=MAX(T,R)  $  SD=T+R-SC END           $
         OTHERWISE                                                               $
BEGIN    SA=MAX(T,R) $  SB=T+R-SA  $  SC=MAX(Q,S)  $  SD=Q+S-SC  END             $
EITHER   IF (SA-SB) LSS (SC-SD)                                                  $
BEGIN    SM = MIN(SA,SB,SC,SD)                                                   $
EITHER   IF ABS(SM-Q) LSS 0.00001                                                $
BEGIN    IKKIR=1.0                          $          SPHIR=0.0                 $
         AG=1       $     MAD=0      $      F=179      $     CA=CCA $  MA=1      $
         ENTER PATH                                                              $
         AG=-1      $     MAD=0      $      F=181      $     CA=CCA $  MA=-1     $
         ENTER PATH                                                   END        $
OR       IF ABS (SM-R) LSS 0.00001                                               $
BEGIN    IKKIR=2.0                          $          SPHIR=0.0                 $
         AG=1       $     MAD=0      $      F=179      $     CA=CCA $  MA=1      $
         ENTER PATH                                                              $
         AG=-1      $     MAD=0      $      F=181      $     CA=CCA $  MA=-1     $
         ENTER PATH                                                   END        $
OR       IF ABS(SM-T) LSS 0.00001                                                $
BEGIN    IKKIR=3.0                          $          GO TO ENE END             $
OTHERWISE                                                                        $
BEGIN    IKKIR=4.0                                                               $
ENE..    SPHIR=ARCCOS((Q.Q+R.R-(S+T)(S+T))/(2.0.R.Q))                            $
         MA=SIGN(W-PPHIR)      $             SPHIR=(-MA)(W(1.0-MA)-SPHIR)        $
         AG=MA      $     MAD=1      $      F=180-MA   $     CA=CCA              $
         ENTER PATH                                                              $
         AG=-MA           $     MAD=0      $           CA=-CA                    $
         IF (D LSS 0.05) AND (DOT EQL 0.0)  $          ENTER PATH                $
         SPHIR=ARCCOS((Q.Q+R.R-(S-T)(S-T))/(2.0.R.Q))                            $
         IF PPHIR GTR W                     $          SPHIR=(2.0.W)-SPHIR       $
         AG=-MA           $     MAD=1      $     F=180+MA   $     CA=CCA         $
         MA=-MA  $  ENTER PATH  $  AG=-MA  $  MAD=0  $  CA=-CA                   $
         IF (D LSS 0.05) AND (DOT EQL 0.0)  $ ENTER PATH END END                 $
OTHERWISE                                                                        $
BEGIN    IKKIR=5.0                                                               $
EITHER   IF ((Q.Q+R.R-(S-T)(S-T))/(2.0.R.Q))  LSS 1.0                            $
         SPHIR=ARCCOS((Q.Q+R.R-(S-T)(S-T))/(2.0.R.Q))                            $
         OTHERWISE                                                               $
         SPHIR=ARCCOS((Q.Q+R.R-(S+T)(S+T))/(2.0.R.Q))                            $
         MA=SIGN(W-PPHIR)      $             SPHIR=(-MA)(W(1.0-MA)-SPHIR)        $
         IF (MA LSS 0) AND ((SPHIR-PPHIR) GTR 0) $             MA= 1             $
         IF (MA GTR 0) AND ((SPHIR-PPHIR) LSS 0) $             MA=-1             $
         AG=MA      $     MAD=1      $      F=180-MA   $     CA=CCA              $
         ENTER PATH                                                              $
         AG=-MA           $     MAD=0      $           CA=-CA                    $
         IF (D LSS 0.05) AND (DOT EQL 0.0)  $          ENTER PATH                $
         SPHIR=(2.0.W)-SPHIR                                                     $
         AG=-MA           $     MAD=2      $     F=180+MA   $     CA=CCA         $
         MA=-MA  $  ENTER PATH  $  AG=-MA  $  MAD=0  $  CA=-CA                   $
         IF (D LSS 0.05) AND (DOT EQL 0.0)  $ ENTER PATH END                     $
         UM(1)=ABS(UM(1))+ABS(UM(4))                                             $
         UM(2)=ABS(UM(2))+ABS(UM(5))                                             $
         UM(3)=ABS(UM(3))+ABS(UM(6))                                             $
```

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        Q=Q/UN   $  R=R/UN   $   S=S/UN   $  T=T/UN   $  M=M/UN   $  N=N/UN  $ 252
        IF UM(3) GTR 0.75                                                     $
BEGIN   WRITE($$TL1)          $          WRITE($$AN6,TL6)                     $
        IF IKKIR EQL 1.0                 $          WRITE($$    TL3)          $
        IF IKKIR EQL 2.0                 $          WRITE($$    TL4)          $
        IF IKKIR EQL 3.0                 $          WRITE($$    TL4)          $
        IF IKKIR EQL 4.0                 $          WRITE($$    TL5)          $
        IF IKKIR EQL 5.0                 $          WRITE($$    TL7)          $
        WRITE($$AN10,TL10)                                                    $ 253
        MA=SIGN(180-FAT(4))                                                   $
        FOR F=(FAT(4)+MA,MA,FAT(1)-MA)                                        $
BEGIN   AC   =(2.0)((((DL(F+1)-DL(F))/(NS(F+1)-NS(F)))-                         255
        ((DL(F)-DL(F-1))/(NS(F)-NS(F-1)))))/(NS(F+1)-NS(F-1)))                $ 256
        VF   =(DL(F+1)-DL(F-1))/(NS(F+1)-NS(F-1))                             $ 257
        WRITE($$AN17,TL17)                                                    $ 258
                                                             END   END        $
JOB..                                                        END              $ 259
OUTPUT  AN10(UN,RAT,Q,R,S,T,E,M,N,PPHI,SSI,GGA,GGB,KKI,                         264
        UM(1),DR(1),DR(4),IP(1),IS(1),GA(1),GB(1),
        CH(1),IP(4),IS(4),GA(4),GB(4),CH(4),
        UM(2),DR(2),DR(5),IP(2),IS(2),GA(2),GB(2),
        CH(2),IP(5),IS(5),GA(5),GB(5),CH(5),
        UM(3),DR(3),DR(6),IP(3),IS(3),GA(3),GB(3),
        CH(3),IP(6),IS(6),GA(6),GB(6),CH(6),     UN)                          $
OUTPUT AN6(K,OMGA,BETA,RHO,ALFA)                                              $
OUTPUT  AN17(NS(F),DL(F),VF    ,AC    )                                       $ 268
FORMAT  TL1(B23,      *EVANS APPROXIMATE STRAIGHT LINE MECHANISMS*,
                  W3,W4,W6)                                                   $
FORMAT  TL2(B5,*THIS IS A DOUBLE CRANK MECHANISM*,W0)                         $
FORMAT  TL3(B5,*R IS THE CRANK OF THIS CRANK AND LEVER MECHANISM*,W0)         $
FORMAT  TL4(B5,*T IS THE CRANK OF THIS CRANK AND LEVER MECHANISM*,W0)         $
FORMAT  TL5(B5,*THIS IS A DOUBLE LEVER MECHANISM*,W0)                         $
FORMAT  TL6(B5,*K=*,X6.2,B3,*OMEGA=*,X6.2,B3,*BETA=*,X6.2,B3,
        *RHO=*,X6.2,B3,*ALPHA=*,X6.2,B3,W2)                                   $
FORMAT  TL7(B5,*THIS IS DOUBLE LEVER MECHANISM OF THE SECOND KIND*,W0)$
FORMAT  TL17(B5,X11.8,X17.8,F20.8,F17.8,W0)                                   $
FORMAT  TL10(B5,*NORMALIZED UNIT LENGTH =*,F14.8,B5,*RATIO =*,F14.8,W6,  1
            B5,*LINK LENGTHS AND COUPLER DIMENSIONS*,W2,                      277
          B5,*Q=* ,F14.8,B2,*R=* ,F14.8,B2,*S=* ,F14.8,B2,*T=* ,F14.8,  278
                                                            W2,               279
            B5,*EPSILON =*,X10.5,B3,*SIDE 1 =*,F14.8,B3,*SIDE 2 =*,         280
                                                       F14.8,W2,              281
            B5,*INITIAL ANGLES*,W2,                                           282
             B5,*PHI=* ,X8.3,B3,*PSI=* ,X8.3,B3,*GAMMA A=* ,X8.3,B3,        283
                 *GAMMA B = *,X8.3,B3,*CHI = *,X8.3,W0,
          3(B5,*LENGTH =*,F14.8,B3,*DL =*,F14.8,B3,*DR =*,F14.8,W4,W0,  285
              B5,*CORRESPONDING LIMITS FOR INPUT, OUTPUT, PRESSURE *,
                 *AND COUPLER ANGLES*,W0,
          2(B5,*PHI=* ,X8.3,B3,*PSI=* ,X8.3,B3,*GAMMA A=* ,X8.3,B3,        288
                 *GAMMA B = *,X8.3,B3,*CHI = *,XB.3,W0)),
             B20,      *EVANS APPROXIMATE STRAIGHT LINE MECHANISMS*,          31
                           W3,W4,W6,                                          31A
             B5,*NORMALIZED UNIT LENGTH =*,F14.8,W4,W2,                       32
             B8,*LOCATION*,B9 ,*DEVIATION*,B8 ,*VELOCITY*,                   308
                           B7,*ACCELERATION*,W2)                            $ 309
        FINISH                                                              $ 310
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EVANS APPROXIMATE STRAIGHT LINE MECHANISMS

K= -2.00 OMEGA= 30.00 BETA= 15.00 RHO= -.40 ALPHA= 75.00

THIS IS DOUBLE LEVER MECHANISM OF THE SECOND KIND

NORMALIZED UNIT LENGTH = .25826922, 01 RATIO = .37509824, 01

LINK LENGTHS AND COUPLER DIMENSIONS

Q= .13769198, 01 R= .12297611, 01 S= .52704054, 00 T= .77612121, 00

EPSILON =-158.4490 SIDE 1 = .23231556, 00 SIDE 2 = .74799900, 00

INITIAL ANGLES

PHI= 350.965 PSI= 234.682 GAMMA A= 43.101 GAMMA B = 72.281 CHI = 306.963

LENGTH = .65321808, 00 DL = .12736794,-02 DR = .14060792,-02

CORRESPONDING LIMITS FOR INPUT, OUTPUT, PRESSURE AND COUPLER ANGLES

PHI= 4.000	PSI= -5.103	GAMMA A= 43.101	GAMMA B = 68.801	CHI = -8.584
PHI= -21.000	PSI= 4.619	GAMMA A= .339	GAMMA B = 63.130	CHI = 49.208

LENGTH = .95954087, 00 DL = .11789887,-01 DR = .13645989,-01

CORRESPONDING LIMITS FOR INPUT, OUTPUT, PRESSURE AND COUPLER ANGLES

PHI= 11.000	PSI= -15.039	GAMMA A= 43.101	GAMMA B = 66.830	CHI = -21.406
PHI= -27.000	PSI= -3.234	GAMMA A= .339	GAMMA B = 36.940	CHI = -292.457

LENGTH = .11674425, 01 DL = .50876984,-01 DR = .56251813,-01

CORRESPONDING LIMITS FOR INPUT, OUTPUT, PRESSURE AND COUPLER ANGLES

PHI= 19.000	PSI= -24.531	GAMMA A= 43.101	GAMMA B = 66.830	CHI = -30.428
PHI= -33.000	PSI= -14.517	GAMMA A= .339	GAMMA B = 6.935	CHI = -273.734

EVANS APPROXIMATE STRAIGHT LINE MECHANISMS

NORMALIZED UNIT LENGTH = .25826922, 01

LOCATION	DEVIATION	VELOCITY	ACCELERATION
.74281294	-.03022105	-.94608585, 00	-.54743114, 02
.72133094	-.01987452	-.37442800, 00	-.94105218, 01
.69854429	-.01364598	-.22384779, 00	-.42082692, 01
.67502213	-.00950840	-.14735203, 00	-.23766096, 01
.65099666	-.00663974	-.10112477, 00	-.14976656, 01
.62659100	-.00461081	-.70749225, -01	-.10023212, 01
.60187938	-.00316474	-.49842642, -01	-.69491268, 00
.57691098	-.00213463	-.35061905, -01	-.49181796, 00
.55172064	-.00140607	-.24461454, -01	-.35140322, 00
.52633387	-.00089744	-.16816857, -01	-.25181810, 00
.50077086	-.00054926	-.11312880, -01	-.17941296, 00
.47504747	-.00031724	-.73807514, -02	-.12670846, 00
.44917748	-.00016846	-.46069712, -02	-.87992164, -01
.42317322	-.00007826	-.26913491, -02	-.59510720, -01
.39704506	-.00002816	-.13906665, -02	-.40160348, -01
.37080473	-.00000543	-.53245226, -03	-.25324446, -01
.34446210	-.00000016	.11004816, -04	-.15979559, -01
.31802881	-.00000601	.35395380, -03	-.99938066, -02
.29151696	-.00001890	.57019033, -03	-.63326914, -02
.26493908	-.00003628	.71079402, -03	-.42556676, -02
.23831290	-.00005671	.80733573, -03	-.29999628, -02
.21165236	-.00007930	.87195175, -03	-.18501395, -02
.18497769	-.00010322	.89615323, -03	.34605214, -04
.15931011	-.00012711	.84773619, -03	.36007470, -02
.13167369	-.00014541	.69064416, -03	.89606944, -02
.10809530	-.00016333	.32887518, -03	.17547930, -01
.07960459	-.00016586	-.29738303, -03	.29810612, -01
.05223408	-.00014761	-.13068225, -02	.46889518, -01
.02601980	-.00009714	-.28255192, -02	.69212416, -01
.00000000	-.00000002	-.50013100, -02	.98398664, -01
-.02578518	.00016194	-.80054864, -02	.13517742, 00
-.05129271	.00041059	-.12023477, -01	.18067320, 00
-.07647854	.00077145	-.17259959, -01	.23628920, 00
-.10129744	.00127367	-.23943733, -01	.30386960, 00
-.12570397	.00195009	-.32311349, -01	.38387010, 00
-.14965352	.00283612	-.42612708, -01	.47908230, 00
-.17311206	.00396985	-.55120257, -01	.59116964, 00
-.19600709	.00539119	-.70110682, -01	.72202742, 00
-.21833233	.00714098	-.87873150, -01	.87467824, 00
-.24003931	.00926037	-.10871807, 00	.10517434, 01
-.26109658	.01178968	-.13202584, 00	.12562176, 01
-.28147630	.01476842	-.16084511, 00	.14933427, 01
-.30115431	.01823297	-.19280882, 00	.17668297, 01
-.32011116	.02221756	-.22916749, 00	.20828536, 01
-.33833118	.02675270	-.27071058, 00	.24497750, 01
-.35580290	.03186542	-.31666590, 00	.28761630, 01
-.37251810	.03757853	-.36871198, 00	.33748060, 01
-.38847180	.04391063	-.42700568, 00	.39616952, 01

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